Aisle Configurations for Unit-Load Warehouses

Kevin R. Gue
Department of Industrial & Systems Engineering
Auburn University
Auburn, Alabama 36849
kevin.gue@auburn.edu

Russell D. Meller
Department of Industrial Engineering
University of Arkansas
Fayetteville, Arkansas 72701
rmeller@uark.edu

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Abstract

Unit-load warehouses are used to store items—typically pallets—that can be stowed or retrieved in a single trip. In the traditional, ubiquitous design, storage racks are arranged to create parallel picking aisles, which force workers to travel rectilinear distances to picking locations. We consider the problem of arranging aisles in new ways to reduce the cost of travel for a single-command cycle within these warehouses. Our models produce alternative designs with piecewise diagonal cross aisles, and with picking aisles that are not parallel. One of the designs promises to reduce the expected distance that workers travel by more than 20 percent for warehouses of reasonable size. We also develop a theoretical bound that shows that this design is close to optimal.

1 Warehouse Design

A distribution center consists of several component subsystems, including receiving, storage, order picking, and shipping. Perhaps the most common building block in these systems is the pallet storage area, which consists of storage racks, aisles between them, and one or more pickup and deposit (P&D) points. In the academic literature this area is commonly called a "warehouse," and we adopt that terminology here. Because almost all products are received and stored in pallet quantities, pallet warehouses tend to consume the majority of space within a distribution center.

Warehouses in industry are typically comprised of single- or double-deep pallet racks arranged in parallel picking aisles, as in Figure 1. In *order picking warehouses*, workers travel through aisles with picking carts (or perhaps ride forklifts, with empty pallets) and build orders by picking items or cases from the stored pallets. Large order picking warehouses usually have one or more *cross aisles* (Figure 1, right), which tend to reduce the travel distance between successive picks in a tour (Roodbergen and de Koster, 2001).

In *unit-load warehouses*, which are the subject of our work, items are stowed and retrieved in pallet quantities, and each stow or pick is for a single pallet. Unit-load warehouses are used in at least two ways in a distribution center: (1) as order picking areas, where products are received and shipped in pallet quantities (distributors of groceries or appliances are two examples); and (2) as reserve areas that replenish fast-pick areas (Bartholdi and Hackman, 2008). For example, a common arrangement is to have a fast-pick "module," consisting of a gravity-fed, carton flow rack, replenished from a pallet reserve area. Order pickers build detailed orders from the flow rack, while workers on lift trucks replenish product into the flow rack from the reserve pallet area. Unit-load operations are also common in crossdocking, where pallets are stored briefly before being loaded onto outbound trucks.

Unit-load warehouses may use single-command cycles, dual-command cycles, or both. In a single-command cycle, a worker accomplishes either a stow or a pick in each trip into the warehouse. Therefore, travel is to and from a single storage location. In a dual-command cycle, often referred to as "task interleaving," a worker visits two storage locations per trip

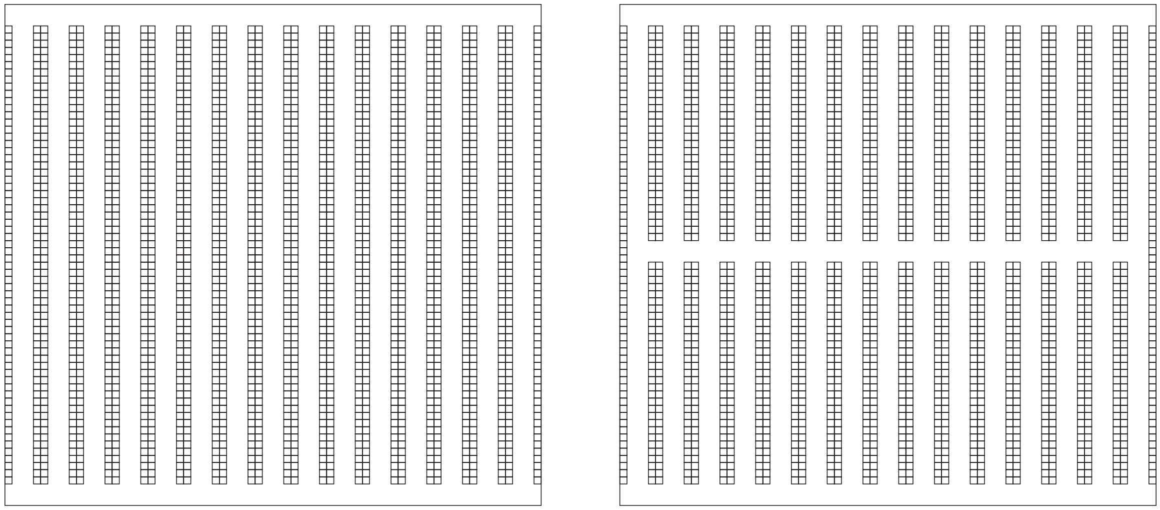

Figure 1: Configurations of typical warehouses in industry.

into the space—first for a stow, then for a pick. Dual-commands are more efficient with respect to travel, but they require concurrent receiving (putaway) and shipping (picking) operations and advanced IT systems to direct workers to picks. As a result, single-command operations are common.

Whether a unit-load warehouse uses single- or dual-commands determines, to some extent, the design of the storage space. If operations are exclusively single-command, then a traditional middle aisle (see Figure 1) confers no benefit. It is easy to see why: if the P&D point is along the lower boundary of the picking space, inserting a cross aisle moves approximately half the locations further away. There is no corresponding benefit because the travel distance to every location is still the rectilinear distance from the P&D point. If operations are exclusively dual-command, then a middle aisle can be beneficial, depending on the size of the warehouse (Roodbergen and de Koster, 2001). Our work addresses only single-command operations in unit-load warehouses.

We believe that all, or nearly all, unit-load warehouses in industry conform to two *Unspoken Design Rules*:

1. Picking aisles must be straight, and parallel to one another.

2. If present, cross aisles must be straight, and they must meet picking aisles at right angles.

Given these two rules and a rectangular warehouse space, the designs in Figure 1 are probably the only options (large warehouses may have more than one middle aisle). Yet both designs tend to limit productivity in a single-command unit-load warehouse. This leads us to ask,

How should cross aisles and picking aisles be arranged to minimize the expected distance to pick in a single-command unit-load warehouse?

We answer this question with two design models, which produce designs that, to our knowledge, were not found in industry before our work, but that offer significant reductions in expected travel distances over traditional designs.

2 Literature review

The aisle design problem is the first of three related problems in warehouse design. The second is product allocation, which seeks to put products in the right locations. The third is the order picker routing problem (and order batching problem, if appropriate), which determines the best sequence of locations for a worker to visit when building orders.

Fundamental relationships for the length and width of a rectangular warehouse are in Tompkins et al. (2003) and Heragu (2006). Francis (1967) investigated rectangular warehouse shapes to minimize picking and construction costs. He assumed rectilinear travel paths, which "presupposes that there is an orthogonal network of aisles running parallel to the x and y axes." Bassan et al. (1980) developed models to determine when it is best to align picking aisles horizontally or vertically in a warehouse, but they assume the traditional structure of Figure 1, with all picking aisles parallel. Berry (1968) noticed that floor-stored pallets should be arranged in lanes with different depths, based on demand characteristics for the SKU, and that different lane depths can be arranged to form "diagonal gangways" in the storage space. He did not explore the implications of this observation.

Especially relevant to our work is White (1972), which proposes non-rectangular warehouses with two or more “radial aisles” projecting away from a single P&D point. His goal was to “approximate Euclidean efficiencies.” Radial aisles are similar to the “fishbone” designs we propose below, however our work is different than White’s in a number of ways: (1) White’s model is descriptive, taking as input a particular design and producing as output expected travel distance. Our models are prescriptive: they take as input some system characteristics, such as the distance between aisles and the length and width of the space, and they produce as output an aisle design that minimizes expected travel distance. (2) Our designs adhere to the industry standard of rectangular picking spaces, which we believe makes them more likely to be implemented in practice. (3) We model the picking space as a set of discrete aisles, whereas White models it as a continuous space. This makes our model slightly more accurate. And (4), our models account for the width of the cross aisle and its effects on travel distances; White assumes aisles have zero width. This is an important difference because, in our experience, the first objection of managers to new aisle designs is the effect of cross aisles on storage density. We should also add that White does not address the best shape for a single cross aisle when picking aisles are parallel, which we describe below.

The warehouse aisle design problem is similar in principle to street design in an urban area. Arlinghaus and Nystuen (1991) mention the effect of a diagonal link in an otherwise rectangular grid network, but they do not offer a model. Their concern is the interaction between pedestrians and automobiles. A famous example of “diagonal travel” in an urban setting is Broadway in Manhattan, which it affords a benefit over traveling streets and avenues exclusively.

Product allocation problems in warehouses are of two main types: allocating products among areas in a warehouse, and allocating products to locations within those areas. The first type includes work on the forward-reserve problem (e.g, Bartholdi and Hackman, 2008; Hackman and Rosenblatt, 1990) and more general product allocation models (Heragu et al., 2005). Product allocation problems of the second type are based on the well-known cube-per-order index rule (Heskett, 1963; Kallina and Lynn, 1976), which assigns products with the highest activity per location to the best locations. In a warehouse like the one that we study, this leads to storing the "fast movers" in a triangular pattern around the P&D point (Francis et al., 1992; Tompkins et al., 2003).

Operationally, product allocation models assume some form of dedicated storage, which reserves particular locations for particular SKUs. Dedicated storage is common in order picking warehouses because such a policy tends to reduce labor costs, which is the primary design concern. In unit-load warehouses, dedicated storage is less common because it tends to reduce storage density, which is a primary design concern. Many unit-load warehouses—most, in our experience—use a storage policy in which any product may occupy any location, depending on availability at the time of storage (e.g., "closest-open location" or randomized policies). Our models assume randomized storage, which was shown by Schwarz et al. (1978) to approximate closest-open location.

The seminal work in routing order pickers is by Ratliff and Rosenthal (1983), who showed that routing a worker to pick several items from a rectangular order picking warehouse (Figure 1, left) is a solvable case of the Traveling Salesman Problem. Roodbergen and de Koster (2001) extended Ratliff and Rosenthal's results to the case of rectangular picking areas with one or more cross aisles (Figure 1, right) in the middle of the picking space. They found that having a "middle aisle" is not beneficial when retrieving a single item, for reasons we have already discussed. For "reasonably-sized" pick-lists, a cross aisle allows shorter tours. For very large pick lists, which are not uncommon in industry, the cross aisle again confers a *dis*-advantage because nearly every aisle is traversed in its entirety and the cross aisle effectively makes the picking aisles longer. Vaughan and Petersen (1999) used heuristic routing techniques to determine the number of cross aisles in a picking area.

With the exception of White (1972), and possibly Berry (1968), the warehouse literature uniformly addresses the operation of warehouses with a presupposed design, which is always some variation of the designs in Figure 1. The presupposition is understandable, because such designs are so commonly found in practice: We have presented our ideas to more than

200 warehouse managers, engineers, and logistics executives, and, before our work, none had ever seen a warehouse with structure different than those in Figure 1.

Our contribution in this work is four-fold. First, we re-introduce a problem fundamental to the design of warehouses which has lain dormant since the publication of White's technical note more than 30 years ago. Second, we propose a warehouse model capable of representing a cross aisle of any shape, and use it to investigate optimal designs. Third, we demonstrate as incorrect the established orthodoxy that inserting a cross aisle is not beneficial in a unit-load warehouse that uses single-commands. Perhaps it is better to say that we show that an inserted cross aisle of the "right shape" is beneficial, whereas a cross aisle of the traditional shape is not. Fourth, the warehouse designs we describe are easy to implement, and should allow distributors to "leverage" the time savings in two ways: by reducing the labor assigned to complete a set of picks, or by using existing labor to complete the picks more quickly. The former reduces costs, the latter improves service; which is preferred depends on the goals of the distributor.

In the next two sections we consider two design problems. The first is to find the optimal cross aisle for a unit-load warehouse with parallel picking aisles. We show that inserting the cross aisle defined by this problem yields a significant reduction in expected travel distance to make a pick. This design is amenable to retrofitting an existing facility. The second problem is to find warehouse designs with picking aisles that are not parallel. We combine this flexibility with a V-shaped cross aisle to build a warehouse with a "fishbone" aisle structure. The expected travel distances for these designs are as much as 20 percent lower than for equivalent traditional warehouses. This design is better-suited to greenfield designs. In Section 5 we propose a theoretical bound for improvement of any aisle design over a traditional warehouse, and show that the improvement offered by fishbone aisles is very close to the bound. We address implementation issues in the final section.

3 An Optimal Cross Aisle

The objective is to minimize the expected travel distance for a single command cycle in a unit-load warehouse that uses randomized storage. We assume there is a single P&D point at the bottom of the space, in the center. A single P&D point might represent a palletizing machine, a shrink-wrap machine, or the station where workers get their picking instructions. We assume workers operate independently and do not interfere with one another; that is, there is no congestion. In our experience, this is a reasonable assumption because the number of workers in most unit-load warehouses is not large enough to create significant congestion, and typical aisles are wide enough to allow passing. (Congestion in order-picking areas is considered in Gue et al. (2006); Parikh and Meller (2008).)

In the first model, we insert a cross aisle into a traditional warehouse and search for the shape of the cross aisle that best improves expected travel distance to a location. We model the cross aisle as a set of connected line segments, where each segment connects adjacent picking aisles. The objective is to find points of intersection between the cross aisle and the picking aisles that minimize the expected distance to make a pick or stow. We assume that all picking aisles are parallel (Unspoken Design Rule 1), but do not require that the cross aisle be orthogonal to the picking aisles, or even that it be straight (Unspoken Design Rule 2).

3.1 Model

As represented in Figure 2, consider a set of picking aisles served from a single, centrally-located P&D point, with each aisle having a continuous and uniform distribution of picking activity within the aisle. For large warehouses, continuous picking activity is a reasonable assumption because picking aisles are often more than 50 picking locations long, and we are interested only in the expected distance to store or retrieve a pallet. The uniform distribution of activity is in keeping with the randomized storage policy.

We assume the picking space is symmetric about the P&D point, so we focus on mod-

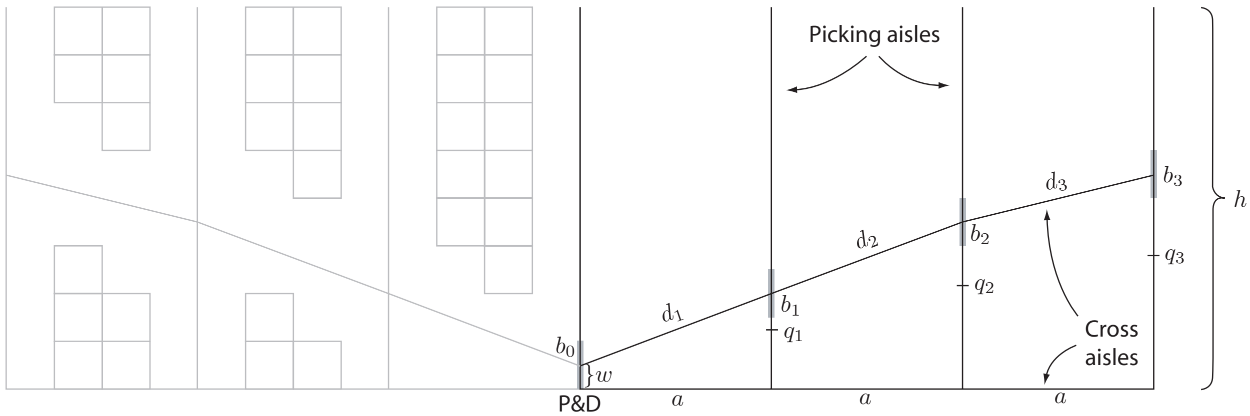

Figure 2: A representation of the continuous optimal cross aisle warehouse model, with nominal rack locations shown on the left side.

eling a picking half-space only. The goal is to construct a cross aisle in the half-space that minimizes the expected distance to make a pick. The P&D point is located in the bottom left corner of the half-space, when viewing it from the top. The cross aisle intersects the line that represents each picking aisle i at a point b_i, including picking aisle zero, which ends at the P&D point. A line connecting the points b_i forms the cross aisle (see Figure 2).

We assume the cross aisle consumes $2w$ of each picking aisle (indicated by the gray boxes in Figure 2), and so inserting it causes the storage space to be larger than it otherwise would be. Notice that if the cross aisle does not intersect picking aisles at a right angle, the width of the cross aisle is less than $2w$—in fact, the width depends on the choice of the b_i's. We ignore this detail, and assume a judicious choice of w, so that the resulting cross aisle is sufficiently wide. In practice, one could easily adjust the choice of w and run the model again to achieve a satisfactory width.

Assume the picking half-space has $n + 1$ picking aisles, each with height h, at a distance a apart (aisle zero extends directly upward from the P&D point). In practice, the picking half-space would include a cross aisle of width $2w'$ at the bottom of the picking space and perpendicular to the picking aisles. Because every pick regardless of its location would include w' travel distance across this aisle, we do not consider it in the optimization model. We do include this detail in numerical results on expected travel distance and warehouse space later in the paper. The warehouse area with an inserted cross aisle is $(2n+1)a \times (h+2w')$ and for a traditional warehouse the area is $(2n+1)a \times (h-2w+2w')$.

By Pythagoras' Theorem, the portion of the cross aisle between picking aisles $i-1$ and i has length

$$d_i = \sqrt{a^2 + (b_i - b_{i-1})^2}. \tag{1}$$

We do not require $b_i > b_{i-1}$, and so the model is free to construct a cross aisle taking on any continuous path. Although d_i is a function of decision variables b_i and b_{i-1}, for ease of presentation we choose not to use the more formal $d_i(b_i, b_{i-1})$.

Let $q_i < b_i$ be the point along picking aisle i at which a worker is indifferent to traveling either along the bottom cross aisle (hereafter, bottom-aisle) and up the picking aisle, or along the cross aisle and down (see Figure 2). That is,

$$ia + q_i = b_0 + \sum_{k=1}^{i} d_k + (b_i - q_i),$$

or,

$$q_i = \frac{1}{2}\left(b_0 + \sum_{k=1}^{i} d_k + b_i - ia\right). \tag{2}$$

We require $q_0 = 0$, by definition. Again, q_i is a function of $b_0, b_1, \ldots, b_i$, but we choose the simpler q_i for clarity of presentation.

Let $C_i(y, \vec{b})$ be the travel distance (hereafter, travel cost) to pick an item at distance y from the bottom of aisle i, given a vector $\vec{b} = \{b_0, b_1, \ldots, b_n\}$ and assuming the cross aisle consumes distance $2w$ of each picking aisle. We divide the picking aisle into three regions, corresponding to different travel paths for shortest retrieval. For $y < q_i$, it is best to travel along the bottom-aisle and up. For $q_i \le y \le b_i$, travel is along the cross aisle and down; for $y \ge b_i$, travel is along the cross aisle and then up. We require $w \le b_i \le h - w$ to ensure that each picking aisle retains $h - 2w$ of picking length.

Since we assume a uniform pick density, the expected travel cost for a pick in aisle $i \ge 1$ is

$$E[C_i(\vec{b})] \quad = \quad \frac{1}{h-2w}\int_0^h C_i(y, \vec{b})\, dy$$

$$
\begin{aligned}
&= \frac{1}{h-2w}\left[\int_0^{q_i} C_i(y,\vec{b})\,dy + \int_{q_i}^{b_i-w} C_i(y,\vec{b})\,dy + \int_{b_i+w}^{h} C_i(y,\vec{b})\,dy\right] \\
&= \frac{1}{h-2w}\left[\int_0^{q_i} (ia+y)\,dy + \int_{q_i}^{b_i-w}\left(b_0 + \sum_{k=1}^{i} d_k + b_i - y\right)dy + \right. \\
&\quad \left. \int_{b_i+w}^{h}\left(b_0 + \sum_{k=1}^{i} d_k + y - b_i\right)dy\right] \\
&= \frac{1}{h-2w}\left[q_i\left[ia + \frac{1}{2}q_i\right] + (b_i - w - q_i)\left[b_0 + \sum_{k=1}^{i} d_k + \frac{1}{2}(b_i + w - q_i)\right] + \right. \\
&\quad \left. (h - b_i - w)\left[b_0 + \sum_{k=1}^{i} d_k + \frac{1}{2}(h - b_i + w)\right]\right],
\end{aligned}
$$

where d_i and q_i are given in (1) and (2), respectively. The expression has the following interpretation: if $xy \leq q_i$, the expected travel is along the bottom ia units, then up $q_i/2$ units; if $q_i \leq y \leq b_i - w$, expected travel is up to b_0, along the cross aisle $\sum_{k=1}^{i} d_k$ units, then down $w + (b_i - w - q_i)/2$ units; if $y \geq b_i$, travel is up to b_0, along the cross aisle $\sum_{k=1}^{i} d_k$ units, then up $w + (h - b_i - w)/2$ units. Each expected travel distance is weighted by the length of the appropriate region.

The expected travel cost to pick an item in aisle zero is different because there is no travel along the cross aisle. For $b_0 \geq w$, we have two possible regions—one below the cross aisle, and one above. For a uniform picking density, the expected travel cost is simply the weighted sum of costs to make a pick from the center of each region.

$$
E[C_0(\vec{b})] = \frac{b_0 - w}{h - 2w}\left(\frac{1}{2}(b_0 - w)\right) + \frac{h - b_0 - w}{h - 2w}\left(b_0 + w + \frac{1}{2}(h - b_0 - w)\right) = \frac{h^2 - 4b_0 w}{2(h - 2w)}.
$$

The expected travel cost for a pick in the full picking space includes a term for a pick in aisle zero, plus (due to symmetry) two times the terms for remaining aisles in the half-space,

$$
E[C(\vec{b})] = p_0 E[C_0(\vec{b})] + 2\sum_{i=1}^{n} p_i E[C_i(\vec{b})],
$$

where $p_i = 1/(2n+1)$ is the probability of the pick being in aisle i, for $i = 0, \ldots, n$.

One final detail: if $q_i > b_i - w$, the cost expression $E[C_i(\vec{b})]$ is malformed, and so we require $q_i \leq b_i - w$. This constraint is tight only if w is large with respect to h, or if b_i is very small, neither of which is true for practical problems. We can restate this constraint as

$b_i \geq w + q_i$, which is stronger than the previous constraint $b_i \geq w$. The optimization problem is to choose $\vec{b} = \{b_0, \ldots, b_n\}$ such that $E[C(\vec{b})]$ is minimized, subject to $w + q_i \leq b_i \leq h - w$, for all i.

Note that we have chosen not to formulate the problem with a network model, with arcs connecting points between adjacent aisles and a "shortest path" representing the cross aisle. Doing so would have been appealing for two reasons: (1) it might be easy to solve to optimality, and (2) a warehouse full of pallet rack imposes discrete locations for aisles anyway. Unfortunately, this approach is not possible because we cannot assign *a priori* costs to arcs that would make up the cross aisle. For example, the cost of an arc should represent the level of flow along the arc, but that flow cannot be specified until the flows along the cross aisle beyond that arc are specified. Similarly, the flow along an arc cannot be specified until the aisle arcs *before* it are specified, because they determine how costly it is to get to that arc. Because costing the arcs requires that the entire cross aisle be specified *a priori*, the problem does not lend itself to constructive graph algorithms or to a sequential optimization technique such as dynamic programming.

We use instead numerical, nonlinear optimization techniques to solve our continuous formulation, and it solves easily. We applied a number of standard algorithms in MATHEMATICA and found no measurable difference among them. Although we cannot prove optimality for our solutions, their structure is intuitive and, we believe, likely optimal or close to it. Moreover, it is likely that, in practice, any solution would have to be adjusted anyway to account for the discrete nature of pallet locations.

3.2 Structure of solutions

There is a value of w greater than which we would choose not to have a cross aisle, because its presence pushes some locations so far from the P&D point that the additional distance overwhelms the advantage of having the cross aisle. Unfortunately, that "critical value" of w depends on the vector $\vec{b}$, which in turn depends on w, and so we are unable to compute it directly. Fortunately, the model handles this difficulty implicitly: if w is greater than the critical value, the cross aisle is along the top of the picking space, effectively making it no cross aisle at all.

Analytical results on the structure of optimal solutions are difficult because of the interdependency between the vector $\vec{b}$ and the critical value of w for which we should choose not to have a cross aisle. However, we can show that

Proposition 1 *If $w = 0$, an optimal solution has $b_0 = 0$; that is, the optimal cross aisle begins at the P&D point.*

Proof. By contradiction. Assume there exists an optimal cross aisle in a picking half-space that intersects aisle 0 at a point $b_0 > 0$. Any pick that uses the cross aisle requires $b_0 + \sqrt{a^2 + (b_1 - b_0)^2}$ travel to get to the first aisle, which is greater than $\sqrt{a^2 + b_1^2}$ by the triangle inequality; that is, letting $b_0 = 0$ reduces the expected distance to any pick using the cross aisle, without changing the distance to any pick not using it. Therefore, the cross aisle is not optimal, a contradiction. □

The result agrees with our intuition that the purpose of a cross aisle is to get the worker into the interior of the picking space as quickly as possible.

Examples. Assume that with appropriate clearances a pallet occupies a $4' \times 4'$ square, and picking and cross aisles are 10 feet wide. These values will vary in practice, depending on pallet sizes, type of picking vehicles, and so on, but our choices represent the majority of warehouses in industry (Tompkins et al., 2003). In what follows, our unit of measure is a pallet; e.g., aisles are 2.5 pallets wide, and perhaps 50–100 pallets long.

Figure 3 shows solutions to our model for two warehouses, one with picking aisles 100 pallets long, the other 50. Due to the shape of the cross aisle, we refer to this design as the "Flying-V." The upper design has 21 picking aisles, the lower has 41 aisles. Distance between centers of picking aisles is equivalent to 4.5 pallets (about 18 feet). Dots in the figure correspond to b_i values from the model. The upper design has expected travel distance to a pick 10.0 percent lower than a traditional warehouse (Figure 1, left) with the same

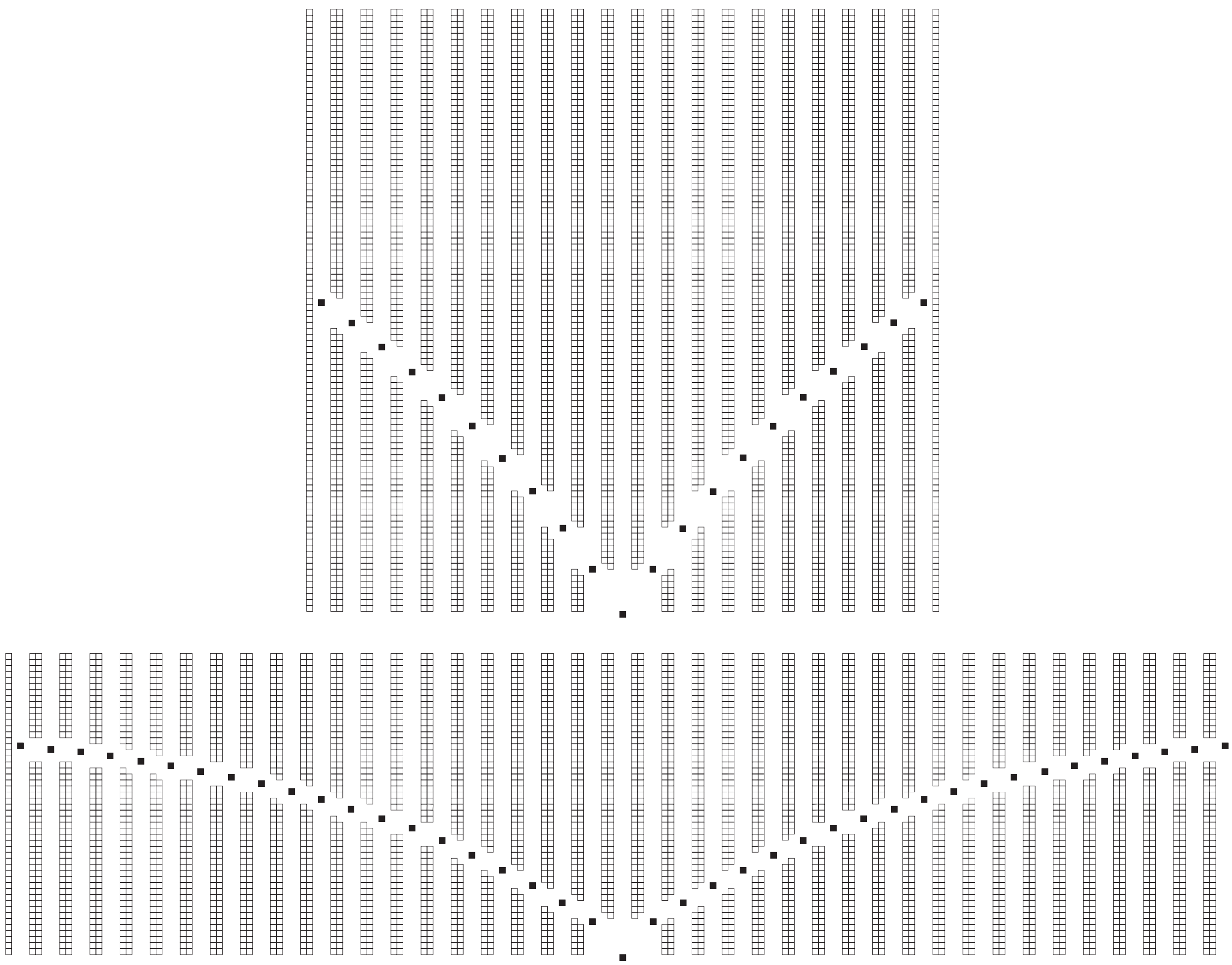

Figure 3: Results of the model for two random-storage, unit-load warehouses with picking aisles 100 and 50 pallets long.

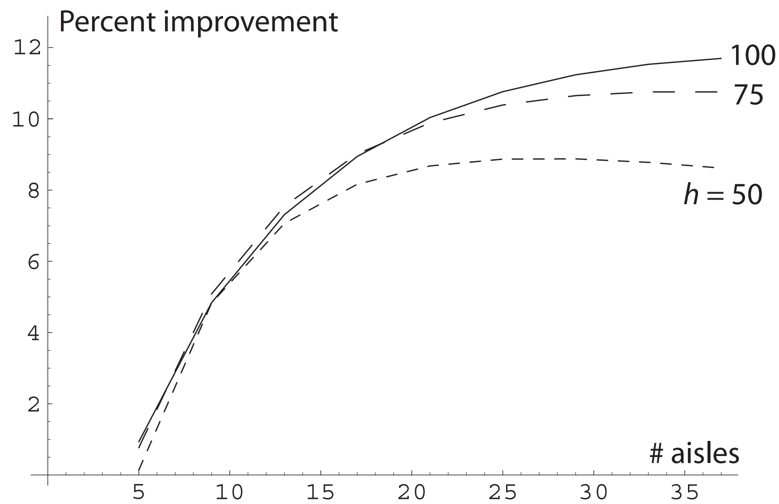

Figure 4: Percent improvement for warehouses with a Flying-V cross aisle, compared to a traditional warehouse.

number and total length of picking aisles. The lower design offers an advantage of 8.4 percent. These comparisons keep the picking positions constant, but increase the size of the warehouse to account for inserting a cross aisle (the Flying-V warehouse spaces are 4.1% and 8.5% larger, respectively, than in comparable traditional designs). The main insight behind these designs is that a cross aisle that cuts diagonally across the picking aisles affords "Euclidean efficiencies" White (1972), which allows workers to get to most picking locations more quickly. Workers would prefer strict rectilinear travel only to locations near the bottom of the warehouse.

Figure 4 illustrates the advantage of Flying-V cross aisles for warehouses of several sizes. In general, as the number of aisles increases, the advantage of inserting a cross aisle increases, until the warehouse is so large that travel to outlying points is dominated by travel *to* the picking aisle rather than within the picking aisle. For warehouses this large, travel along the cross aisle is almost equal to travel along the bottom aisle, and little advantage is gained from having the cross aisle. Unit-load warehouses in industry typically have 20–40 aisles, which is the range for which this design confers the greatest advantage. Warehouses with longer picking aisles stand to benefit more from Flying-V cross aisles than do those with short picking aisles.

4 Fishbone Aisles

Next, we relax Unspoken Design Rule 2, which requires that all picking aisles be parallel. In the most general case, picking aisles can take on any angle, but here we consider only vertical and horizontal orientations. We also restrict ourselves to a cross aisle that extends in two diagonals away from the P&D point. We show in the next section—through a theoretical lower bound on expected travel cost—that these constraints do not keep us far from an optimal solution.

4.1 Model

Because the picking space is symmetric, we focus again on a picking half-space. Consider a half-space with n vertical aisles, each of height h and separated by distance a. For now, we do not include an aisle extending from the P&D point. Using insights from the previous model, we insert a straight, diagonal cross aisle passing through the P&D point. Let $0 \leq b \leq h - w$ be the point of intersection of the cross aisle with the rightmost (n-th) picking aisle. Using the P&D point as the origin in a coördinate system, we see that the cross aisle has slope $m = b/na$. The case of the diagonal cross aisle having a greater slope, and therefore reaching the top of the warehouse before reaching the rightmost aisle, is easily handled by "inverting" the space (thereby making the new height equal to na) and re-solving. The cross aisle consumes distance w of each picking aisle (see Figure 5).

We break the total expected travel cost into three components: aisle zero, which extends upward from the P&D point, upper aisles (with respect to the cross aisle), and lower aisles. We assume again that picking activity is distributed uniformly among and within all aisles; therefore, expected cost to make a pick in aisle zero is simply

$$E[C_0] = \frac{1}{h-w}\int_w^h y\,dy = (h+w)/2.$$

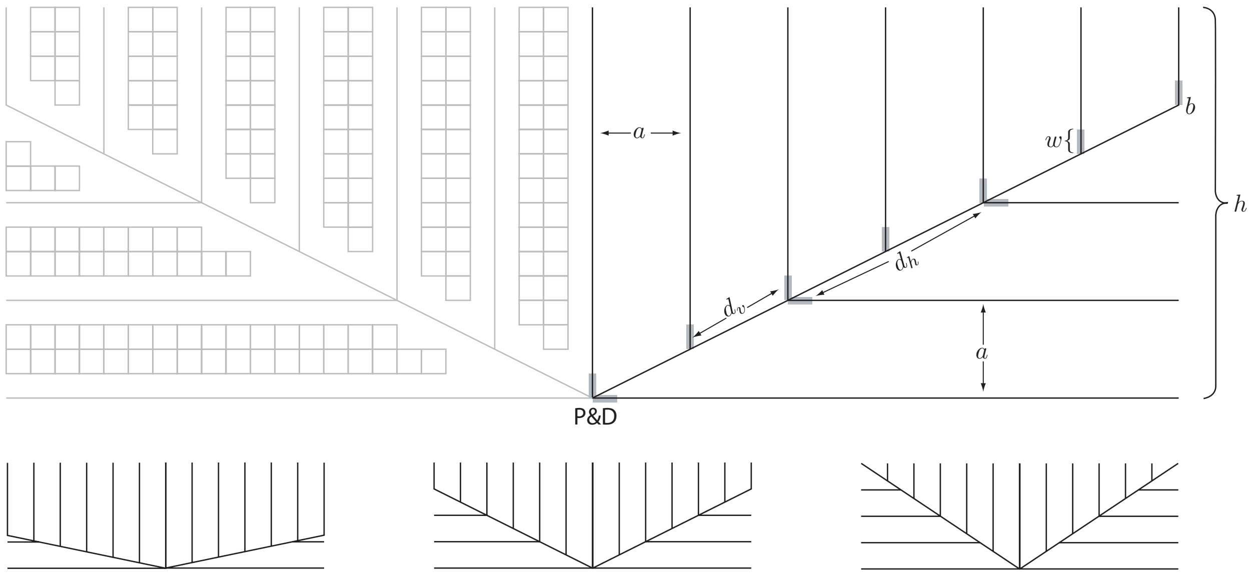

Figure 5: A representation of the continuous model for warehouses with fishbone aisles, with nominal rack locations shown on the left. Figures on the bottom illustrate changes in aisle structure with different values of the parameter b.

The travel cost for picking in upper aisle i, $C_i^u(y, b)$, is the required distance along the diagonal aisle, plus the travel up to point y. Between each vertical picking aisle, the cross aisle has distance (see Figure 5),

$$d_v = \sqrt{a^2 + (am)^2} = \sqrt{a^2 + \left(\frac{b}{n}\right)^2}.$$

The expected travel cost of a pick in an upper aisle is,

$$\begin{aligned} E[C_i^u(b)] &= \frac{1}{h - mia - w} \int_{mia+w}^{h} (id_v + y - mia)\, dy \\ &= id_v + \frac{1}{2}(h - mia + w) \\ &= i\sqrt{a^2 + \left(\frac{b}{n}\right)^2} + \frac{1}{2}\left(h - \frac{ib}{n} + w\right). \end{aligned} \tag{3}$$

For the lower picking area we have,

$$d_h = \sqrt{a^2 + (a/m)^2} = \sqrt{a^2 + \left(\frac{a^2 n}{b}\right)^2}.$$

Lower aisle j begins at position aj/m and ends at position an, so

$$
\begin{aligned}
E[C_j^\ell(b)] &= \frac{1}{an - aj/m - w} \int_{aj/m+w}^{an} (jd_h + x - aj/m)\, dx \\
&= jd_h + \frac{1}{2}(an - aj/m + w) \\
&= j\sqrt{a^2 + \left(\frac{a^2 n}{b}\right)^2} + \frac{1}{2}\left(an - \frac{ja^2 n}{b} + w\right). \qquad (4)
\end{aligned}
$$

Both (3) and (4) agree with the intuition that expected travel cost for a pick in an aisle is travel to the picking aisle plus half the length of the aisle.

Notice from Figure 5 that there are values of b for which the distance between the center of the cross aisle and the end of a potential vertical or horizontal aisle is less than w; that is, there would be effectively no picking aisle at all. For a horizontal aisle, this happens when b is slightly greater than a multiple of a. For a vertical aisle, if $b = h - w$ and h is small with respect to na, the length of the rightmost vertical aisles could be less than w. Computationally, an aisle shorter than w causes (3) or (4) to be negative, and so we define the length of a vertical aisle $l_i^v = \max\{h - mia - w, 0\}$ and for a horizontal aisle $l_j^h = \max\{an - aj/m - w, 0\}$. Note that the number of aisles having non-zero length in both the upper and lower regions is a function of parameter b.

Let U be the index set of picking aisles in the upper region (perhaps with length zero), and L the index set in the lower region. For a uniform picking density, the probability p_i of picking in aisle i equals the length of that aisle divided by the sum of lengths of all picking aisles. For an aisle in the upper region,

$$
p_i^u = \frac{l_i^v}{h - w + 2(\sum_{k\in U} l_k^v + \sum_{j\in L} l_j^h)}.
$$

For an aisle in the lower region,

$$
p_j^\ell = \frac{l_j^h}{h - w + 2(\sum_{i\in U} l_i^v + \sum_{k\in L} l_k^h)}.
$$

Expected travel cost for a pick in a fishbone warehouse is comprised of a term for the center (vertical) aisle, plus two times the terms for upper and lower aisles (to account for

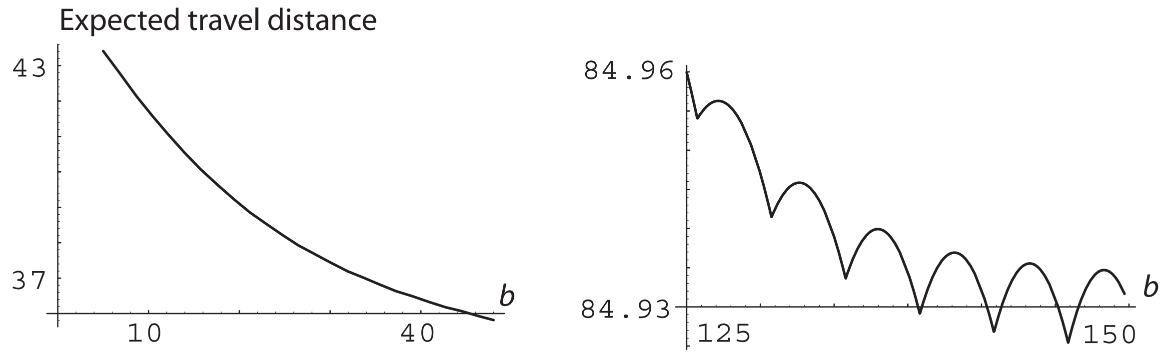

Figure 6: Plots of the objective function as the parameter b varies: the plot on the left represents typical values for aisle length and width; on the right, an example of non-convexity for unrealistic values of parameters. (Note the significant difference in scale of the two plots.)

both half-spaces),

$$E[C(b)] = p_0^u E[C_0] + 2\left(\sum_{i\in U} p_i^u E[C_i^u(b)] + \sum_{j\in L} p_j^\ell E[C_j^\ell(b)]\right). \quad (5)$$

The optimization problem is to choose b such that $E[C(b)]$ is minimized, subject to $0 \le b \le h - w$.

The problem has only one variable, but for some (admittedly unrealistic) values of h, n, and a, the objective function is not convex (see Figure 6). Therefore, we apply numerical nonlinear optimization techniques, as before. In practice, it is easy to run the model, then inspect a plot of the objective function value as a function of b to ensure the solution is not a local optimum. (It would also be possible to require b to take on integer values, and then enumerate the solution space.) We have solved the fishbone design problem for warehouses of many *practical* sizes, and the optimal $b = h - w$ in every case. The cases where the optimal b is slightly less than $h - w$ include warehouses with relatively few, very long vertical aisles and an unrealistically narrow cross aisle. Figure 6 illustrates such a warehouse with picking aisles 150 pallets deep and cross aisles 3 feet wide.

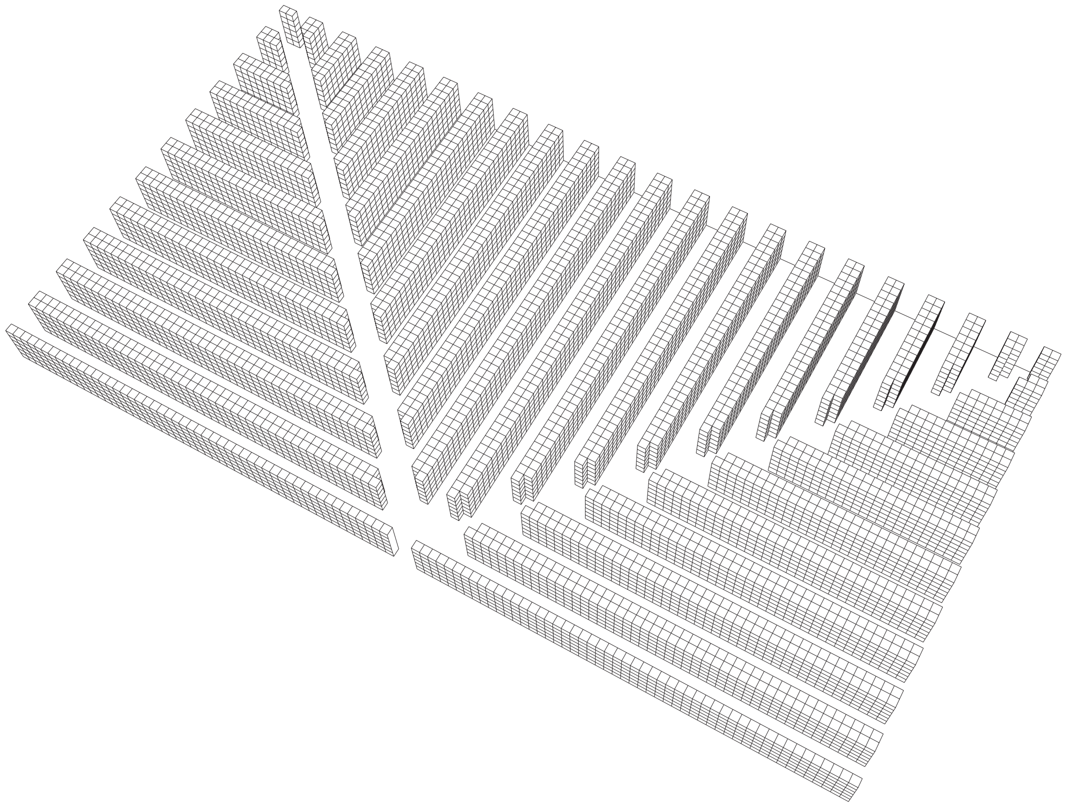

Figure 7: A warehouse with fishbone aisles (note than an implementation of this design in practice would likely clear away some storage racks near the P&D point).

4.2 Example Fishbone Designs

For example solutions, we assume standard pallet sizes and aisle widths as before, and we measure distances in units of pallets. For a fishbone design we must consider the value of w carefully, because picking aisles extend from the cross aisle at different orientations. This is easily adjusted in the model.

Example. Figure 7 shows the solution for the equivalent of 21 vertical picking aisles, each having length of 50 pallet locations. In this case, the optimal $b = h - w$. The intuition is that such designs have a cross aisle that cuts through the “middle” of the picking space, and therefore, because every pick uses the cross aisle, it should be centrally placed.

It is important to recognize that as b increases and the slope of the cross aisle increases,

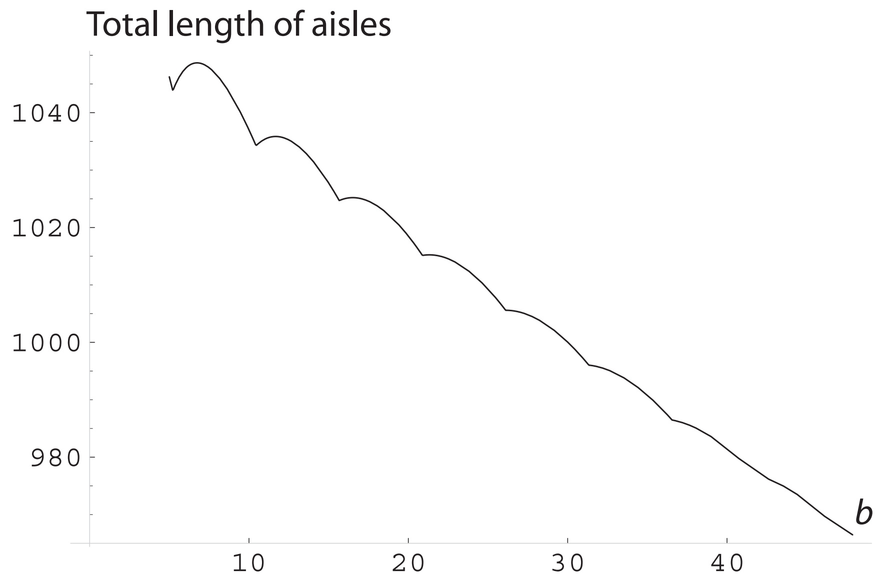

Figure 8: Total length of picking aisles for fishbone designs with $h = 50$ and different values of b.

the total length of picking aisles *decreases* slightly (see Figure 8). Therefore, we must be careful when interpreting a solution. This manifests an important tradeoff between storage density and expected retrieval distance, which we must consider when comparing fishbone designs with a traditional warehouse. To make a fair comparison, we first model a fishbone warehouse, then model a traditional warehouse with the same number of vertical aisles, with the length of the picking aisles adjusted so that the total aisle length equals that of the fishbone design (this also provides for an accurate comparison of the space of each design). The design in Figure 7 has expected travel cost 20.3 percent lower than an equivalent traditional warehouse (although the fishbone warehouse occupies 3.0% more space). Figure 9 shows how the potential travel cost advantage varies for different configurations. In practice, unit-load warehouses range in size between 20 and 40 aisles.

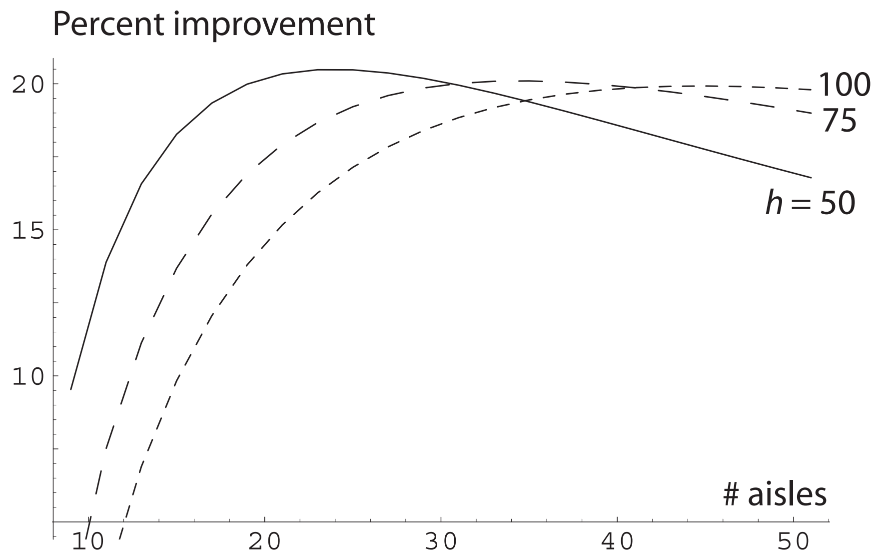

Figure 9: Percent improvement of a fishbone aisle design over a traditional design for several configurations.

5 Bounds

Warehouses with Flying-V cross aisles or fishbone aisles offer a significant potential savings in retrieval time, but how close are they to optimal? Here we offer a lower bound on travel distance based on an imaginary warehouse in which items are distributed uniformly and continuously throughout the picking space, and workers can "fly" directly to and from any location. We use the bound to compute the maximum possible improvement of any design over a traditional warehouse, in which travel is rectilinear. The bounds are similar to well-known results in urban planning, which compare the expected differences between rectilinear travel and Euclidean travel in, for example, an ambulance coverage area (Larson and Odoni, 1981).

Without loss of generality, we can consider a rectangular half-warehouse with dimensions $a \times b$ and a single P&D point in the bottom-left corner. Picking activity is distributed continuously and uniformly throughout the space. We used Mathematica to compute the expected Euclidean distance from a point in the rectangle to the P&D point,

$$\frac{1}{ab}\int_0^a\int_0^b \sqrt{x^2+y^2}\;dy\,dx = \frac{1}{6ab}\left(2ab\sqrt{a^2+b^2}+a^3\log\left(\frac{b+\sqrt{a^2+b^2}}{a}\right)+b^3\log\left(\frac{a+\sqrt{a^2+b^2}}{b}\right)\right).$$

If travel is rectilinear, as in a traditional warehouse, expected distance would be,

$$\frac{1}{ab}\int_0^a\int_0^b (x+y)\;dy\,dx = \frac{1}{2}(a+b).$$

For a square picking half-space with unit-length sides, which has the intuitive appeal of being "balanced" (Francis, 1967, showed this to be an optimal shape, under certain assumptions), an imaginary warehouse has expected distance to make a pick,

$$\int_0^1\int_0^1 \sqrt{x^2+y^2}\;dy\,dx = \frac{1}{3}\left(\sqrt{2}+\sinh^{-1}(1)\right) \approx 0.7652,$$

If travel is rectilinear, as in a traditional warehouse, expected distance is

$$\int_0^1\int_0^1 (x+y)\;dy\,dx = 1.$$

The implication is that, if picking activity is distributed "approximately uniformly" in a square picking half-space, the best possible aisle design can offer no more than $1-0.7652 \approx 23.5$ percent reduction in expected travel distance. A similar result can be derived from results in Francis et al. (1992, Ch. 5): the best possible improvement in a warehouse with a semicircular shape is 24.8 percent. (It is worthwhile noting that a traditional unit-load warehouse—with or without cross aisles—performs *as badly as possible*, when average distance is the performance metric and workers travel to picks along shortest paths.)

The result is different for non-square picking half-spaces. Figure 10 illustrates the benefit of direct, "travel-by-flight" over the rectilinear travel required in a traditional warehouse as a function of the ratio of width to height of the picking space (labeled "Bound"). The benefit has its maximum at 2:1, which corresponds to the commonly found square half-space (and also conforms to the results in Francis, 1967). As the ratio increases, the advantage of direct travel decreases.

The implication is that the potential advantage of better aisle designs is greatest for the warehouse sizes most common in industry. Moreover, the 23.5 percent potential improvement with respect to a theoretical warehouse space suggests that the fishbone design, which

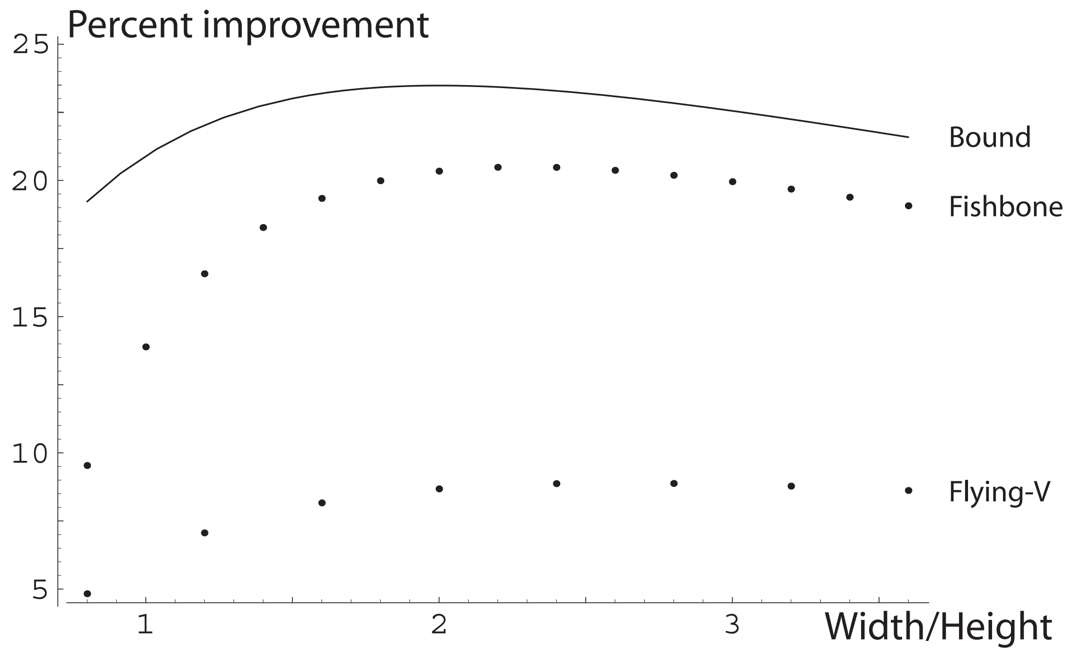

Figure 10: Percent improvement of fishbone and Flying-V warehouses over the traditional design.

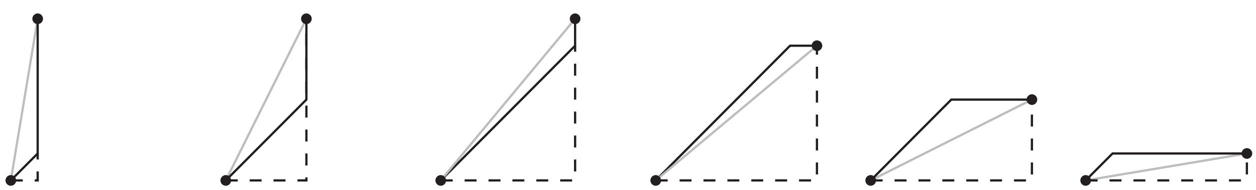

Figure 11: An illustration of fishbone travel (solid black lines) and direct (dashed lines).

showed a 20.3 percent improvement for a square half-warehouse, is close to optimal. Example travel paths represented in Figure 11 show why this is the case: fishbone designs are particularly good for accessing locations near a 45-degree diagonal from the P&D point, the same locations for which rectilinear travel is especially poor.

6 Implementation Issues and Conclusions

Designing a distribution center is a complex task, requiring engineers to design storage areas, order picking areas, and material handling systems with the goal of meeting service requirements at minimum cost. Nearly every distribution center has a unit-load, or pallet, storage area where products are stored before being picked, or before being moved to other order picking areas in the distribution center. We argued that existing warehouses conform to unspoken design rules, which tend to increase travel for workers.

We relaxed two of these design rules and found that there is much to be gained by considering new aisle configurations for unit-load warehouses. Designs we develop in this paper could be useful in any unit-load warehouse with a single, dominant P&D area.

Our results suggest that, for unit-load warehouses, new aisle designs could lead to higher throughput, or to significantly reduced costs of picking. For new warehouses, designers must weigh the value of reduced operating costs of these designs with the fixed cost of needing a slightly larger warehouse. In some situations, bulk areas are seldom visited, and a focus on density rather than retrieval cost is appropriate; but for many applications, we believe labor costs are high enough to justify new designs.

We offer two new designs: The first inserts a "Flying-V" cross aisle into a warehouse with parallel picking aisles. Contrary to the established orthodoxy of unit-load warehouse design, inserting a cross aisle with this shape *reduces* the expected travel distance for workers, and that by about 8–12 percent, depending on dimensions of the warehouse. The second design has "fishbone" picking aisles which extend horizontally and vertically from diagonal ("spine") cross aisles. We showed that this design can reduce expected travel cost by more than 20 percent, and that this is close to optimal.

If fishbone designs offer lower expected retrieval distances than Flying-V designs, why even consider the latter? Because, they have several potential advantages:

- Access into and out of the space is easier, which is a benefit if workers occasionally must make trips to or from points other than the P&D point.

- Forklift traffic is distributed over two cross aisles (diagonal and bottom) in a Flying-V design, unlike in the traditional warehouse and the fishbone warehouse, where all traffic is concentrated along one cross aisle.
- Workers are less likely to become disoriented in a Flying-V warehouse because it is similar to a traditional warehouse. For an industry in which worker turnover is high and experience is low, this can be an important benefit.
- The numbering scheme for picking locations is more intuitive than it would be for a fishbone warehouse.
- It is possible to retrofit an existing traditional warehouse with a Flying-V cross aisle, simply by removing appropriate portions of rack. On this point, we acknowledge the probably insurmountable psychological barrier of paying money to remove "good storage locations." Nevertheless, it is possible, and should be considered if storage capacity allows and there is a significant operational advantage.

We should reiterate that our designs apply only to unit-load warehouses with a single, centrally-located P&D point, and are based on an assumption of random storage. Interesting extensions include models for order picking warehouses, in which workers visit multiple locations per tour, and for warehouses with multiple P&D points and non-uniform storage policies.

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