An Analysis of Dual-Command Operations in Common Warehouse Designs

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September 10, 2008

Abstract

In a warehouse that uses dual-command operations, workers travel loaded from the pickup and deposit (P&D) point first to a location to store a pallet, then to a second location from which they pick a pallet and return to the P&D point. We develop expected travel distance expressions for such operations and use them to analyze three common warehouse designs. Our results indicate that the best of the three is — in our experience — the one least commonly found in practice. We also show that the optimal placement of a "middle cross aisle" in the most common design is, in fact, not in the middle.

1 Introduction

Unit-load warehouses store pallets or other unit-loads of goods in locations commonly arranged in parallel aisles. Examples include third-party transshipment warehouses, beverage and grocery distributors, and appliance manufacturers. Many warehouses that ship in smaller units, such as cartons or pieces, also have a portion of their activity dedicated to unit-loads. For example, the reserve area of a retail distribution center may be used to receive and store pallets until they are needed to replenish a fast-pick area, when they are then "shipped" to another part of the warehouse (Bartholdi and Hackman, 2007).

Unit-load warehouses are especially important in global retail supply chains, where import distribution centers (DCs) near ports are prevalent and large. For example, within a 125 mile radius of the Port of Savannah, the import DCs of just 23 retailers comprise more than 20 million square feet of warehouse space, including 4 warehouses of more than 2 million square feet each (Foltz, 2007). These warehouses are comprised almost entirely of pallet storage areas, which consist of floor storage, in which pallets are stacked directly on top of one another in parallel aisles, and single- or double-deep pallet racks. Because these DCs are so large, the distances traveled to perform operations are also large.

Two features of a unit-load warehouse are of interest to us. The first is the operational protocol for workers storing and retrieving pallets. In single-command operations, workers travel from a pickup and deposit (P&D) point to a single location in the warehouse, where they store or retrieve a single pallet before returning. One half of their travel is unloaded, and therefore unproductive. A second protocol is to interleave storage and retrieval operations to form a dual-command cycle, in which workers perform a storage operation and then continue directly to a retrieval location before returning to the P&D point. Interleaving reduces the empty forklift travel from half of the total travel distance to about one third. We refer to the travel distance between the storage and retrieval locations as "travel-between."

The second feature of interest to us is the arrangement of storage locations and aisles, which we call the *layout*. In our experience, the three most common layouts are those in Figure 1. Layout A (Figure 1a) has parallel picking aisles and orthogonal cross aisles at each end of the picking aisles. Layouts B and C (Figures 1b and 1c) are similar to Layout A, but with a cross aisle inserted halfway along the picking aisles. Layouts B and C can be viewed as the same layout with the P&D point in a different location; but there are reasons to consider them as distinct designs, which we discuss below. In practice, Layouts B and C could also have more than one inserted cross aisle, but we consider only those with a single cross aisle, which are most common by far.

Layout A is a better choice that Layout B for a unit-load warehouse that performs strictly single-command cycles. Roodbergen and de Koster (2001b) point out that if the number of picking locations is fixed, inserting a middle cross aisle increases the expected travel to a single pick because it pushes half the locations farther from the P&D point. However, they show that for "practically-sized" picklists, a middle cross aisle reduces travel distances because it creates more possible routes for a picking tour. The benefit of the middle cross aisle is eliminated when the size of the order is large with respect to the warehouse size, because the worker tends to traverse every picking aisle and not to use the cross aisle at

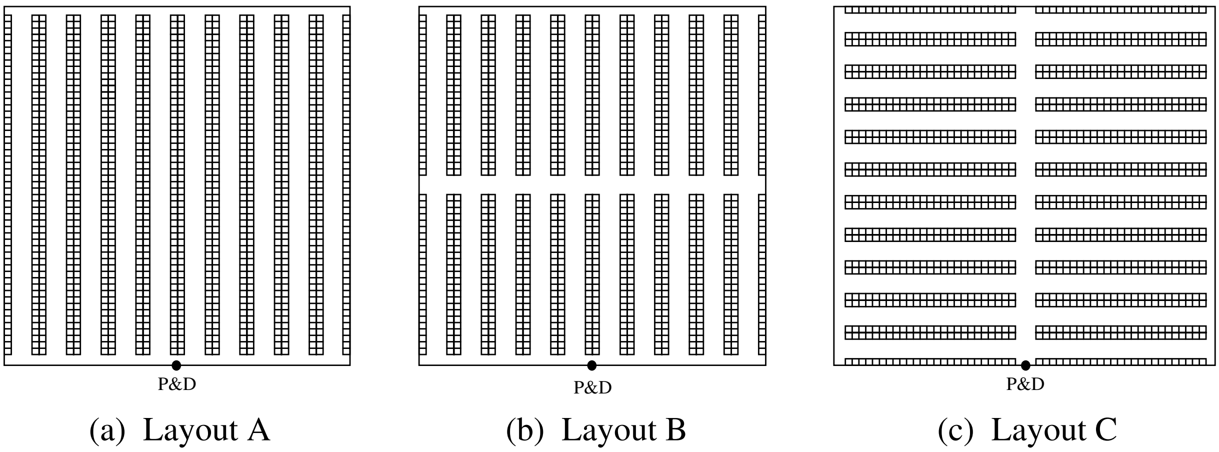


Figure 1: Warehouse Aisle Layouts

all (Roodbergen and de Koster, 2001b). Therefore, Layout B is a potentially better design for dual-command operations. And when considering the choice between Layouts A and C, we will show that if Layout C is configured properly, Layout A no longer has an advantage in terms of single-command cycles. Furthermore, due to the inserted cross aisle, Layout C retains its potential improvement for dual-command operations.

Our goal is to determine which of these three common layouts is best for dual-command operations. To do so, we develop analytical dual-command travel distance models for optimal paths in all three layouts. With these models, we show that the determination of which layout is best depends on the required number of storage locations, but that Layouts B and C are superior to Layout A in almost every case. Layout C is the best choice or near-best choice for a wide range of sizes, yet—in our experience—it is the least popular in practice. Along the way, we investigate the best number and length of picking aisles for a required number of storage locations. We also show that the optimal placement of the "middle aisle" in Layout B is not exactly in the middle, but slightly above it.

In the next section we review what is known about the three designs, mostly in the context of single-command operations. We also examine the literature on dual-command operations. In Section 3 we develop analytical expressions for travel-between distance, which we use in Section 4 to investigate the optimal number and length of aisles for layouts under both single- and dual-command travel. In Section 5, we compare the performance of the three designs for dual-command travel and show when it is best to have a middle aisle. We offer concluding remarks in Section 6.

2 Previous Research

Researchers have modeled single-command travel distance in Layout A (Francis, 1967; Bassan et al., 1980) and Layout C (Bassan et al., 1980) and have presented some well-known results on optimal warehouse shape and P&D location. The few papers that model dual-command travel consider only Layout A (Mayer, 1961; Malmborg and Krishnakumar, 1987), and do not use their results to determine warehouse design parameters, such as number and length of aisles. Mayer (1961) also restricts cross aisle travel to the bottom cross aisle, while we assume the cross aisle that provides the shortest path between pallet locations is used. We are unaware of any published analytical models for the optimal dual-command travel distance in Layouts B and C.

Dual-command travel is a special case of the orderpicking problem. For Layout A, Ratliff and Rosenthal (1983) develop an efficient dynamic programming algorithm to determine the optimal pick tour, which provides the shortest travel distance to a given set of pick locations. Roodbergen and de Koster (2001b) extend their algorithm to consider warehouses with a middle cross aisle (and provide numerical results for Layout B). Since these algorithms determine the optimal tour for just one instance, simulation is required to estimate the expected tour length. That is, there is no closed-form evaluation method for the expected length of an optimal tour for a general number of picks.

Hall (1993) develops a lower bound for the optimal tour length in Layout A for a general number of picks, but we have not found it to be very accurate for a small number of picks. With the exception of Hall's bound, the analytical expressions for expected travel distance in orderpicking are based upon routing heuristics, rather than optimal tour generation. Among those who assume a random storage policy, Hall (1993) and Roodbergen and Vis (2006) model travel in Layout A, Le-Duc and de Koster (2007) model travel in Layout C and Roodbergen et al. (2007) consider Layouts A, B, and C. Analytical models for turnover-based storage policies are developed by Hwang et al. (2004) for Layout A and by Caron et al. (2000b) for Layout C.

Simulation is used to estimate expected travel for optimal and heuristic routes for random storage by Petersen (1997) and de Koster and van der Poort (1998) in Layout A, and by Vaughan and Petersen (1999) and Roodbergen and de Koster (2001a,b) in Layout B. Turnover-based storage is considered by Petersen and Schmenner (1999) for Layout A and by Caron et al. (2000a) for Layout C. The expressions we develop are the first that describe optimal dual-command travel in Layouts A, B and C.

Our modeling of travel in Layout A could be considered a special case of a multi-aisle automated storage and retrieval system (AS/RS). In a multi-aisle system, a single crane services multiple aisles, and therefore must transfer from aisle to aisle. Several authors have addressed this problem. For example, Hwang and Ko (1988) model expected single-command and dual-command travel time in such a system, and as is typical, assume the crane travels simultaneously in the horizontal and vertical directions. By ignoring vertical travel in their models, the models can be used to provide an estimate of the expected travel in Layout A. However, because they assume a corner input/output point (we assume the P&D point is optimally located in the bottom center) and restrict cross aisle travel to the bottom aisle for dual-command cycles, their estimate provides an upper bound.

3 Expected Dual-Command Travel Distance Models

In this section we model the expected single and dual-command travel distances in Layouts A, B and C, assuming optimal travel paths beginning and ending at a single, central P&D location. We assume a random storage policy, which approximates the "closest-open-location" storage rule (Schwarz et al., 1978). This type of storage assignment leads to the most efficient use of storage space and is commonly used in unit-load storage, where one of the primary goals is to maximize space utilization.

We are interested in expected horizontal travel distance; therefore, we do not model the acceleration and deceleration of the vehicle, time to load and unload, or vertical travel to upper pallet positions. Workers can travel in either direction in an aisle and can change directions within an aisle. We assume that the picking aisles are sized such that the racks on each side of the picking aisle can be accessed, but that the lateral travel within the aisle is negligible. That is, the travel distance to a given pallet position is the same as the travel distance to a pallet position directly above or directly across the picking aisle. A "location" in the warehouse therefore refers to two columns of pallet positions, one on each side of the picking aisle. For example, a warehouse with 21 picking aisles that are 50 pallets long, would have a total picking aisle length $T = 1050$. This implies that there are 2100 pallet positions on each level of the storage racks. We model the warehouse as a set of discrete picking aisles, with continuous picking activity in each aisle, and picking uniformly distributed within and among all aisles. The storage and retrieval requests are assumed to be independent, and processed on a first-come-first-serve basis.

3.1 Layout A: Picking Aisles Perpendicular to the Front Wall

Layout A, as depicted in Figure 2, has parallel picking aisles that are perpendicular to the front wall, but has no cross aisle inserted into the picking space. The travel paths along the aisles are indicated by solid black lines. Two locations are indicated in black in Figure 2, where the location on the left is a distance x from the bottom of aisle i, and the location on the right is a distance y from the bottom of aisle j. The length of each picking aisle is L and the distance that must be traveled to enter a picking aisle from the cross aisle is v, or half the width of the cross aisle. The warehouse has n picking aisles, where n can be odd or even.

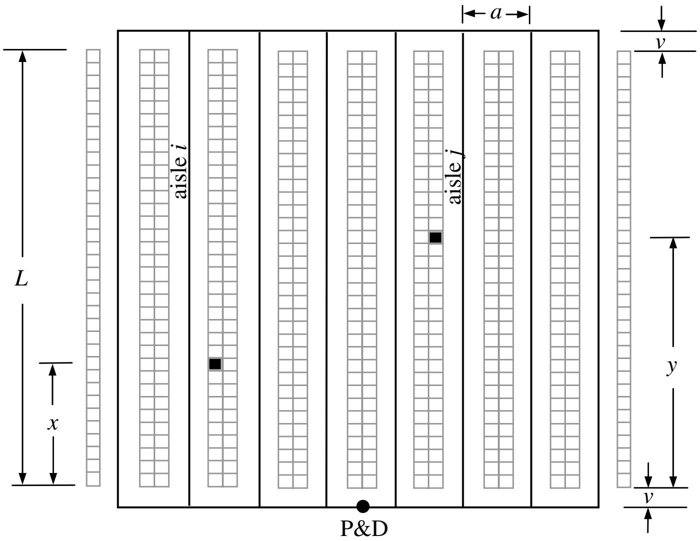


Figure 2: Layout A

The expected dual-command travel distance can be expressed as the sum of the expected single-command travel distance and the expected travel-between distance, $E[DC] = E[SC] + E[TB]$. To determine $E[SC]$, we consider two components of travel: cross aisle travel, and picking aisle travel (including the distance v to enter/exit the picking aisles, if necessary). Since we assume uniform picking activity, the expected picking aisle travel distance in a single-command cycle is $2(L/2 + v) = L + 2v$. To determine the expected cross aisle travel distance, we must assume a P&D location. We assume the P&D point is optimally located in the middle of the bottom cross aisle (Francis, 1967; Roodbergen and Vis, 2006), as shown in Figure 2, and that there is an even number of aisles. The expected cross aisle travel for an even number of aisles is $an/2$. (The expected cross aisle travel for an odd number of aisles is $a(n^2 - 1)/2n$.) Thus, the expected single-command travel distance for Layout A with an even number of aisles is

$$E[SC_{\mathrm{A}}] = L + 2v + \frac{an}{2}. \tag{1}$$

The travel between two locations does not depend on the location of the P&D point or whether the number of picking aisles is even or odd. If the two locations are in the same picking aisle ($i = j$), travel-between distance, or TB, will be $|x - y|$. If the locations are in different aisles, as pictured, travel can use either the top cross aisle or the bottom cross aisle. If the bottom cross aisle is used, then $TB = x + v + a|i - j| + v + y$, where a is the distance between picking aisles. If the top cross aisle is used, $TB = (L-x)+v+a|i-j|+v+(L-y)$. To determine the expected travel distance between two random locations, we again consider two components: cross aisle travel, and picking aisle travel. Since we assume uniform picking activity, the expected cross aisle travel distance is

$$\frac{a(n^2-1)}{3n}, \tag{2}$$

and the expected picking aisle travel distance is

$$\frac{1}{n}\left(\frac{L}{3}\right) + \frac{n-1}{n}\left(\frac{2}{3}L + 2v\right). \tag{3}$$

The derivations of (2) and (3) are found in the appendix. Given (2) and (3), the expected travel distance between locations in Layout A is

$$E[TB_{\mathrm{A}}] = \frac{1}{n}\left[\frac{L}{3} + (n-1)\left(\frac{2}{3}L + 2v\right)\right] + \frac{a(n^2-1)}{3n},$$

and the expected dual-command travel distance with an even number of aisles is

$$E[DC_{\mathrm{A}}] = L\left(\frac{5n-1}{3n}\right) + v\left(\frac{4n-2}{n}\right) + a\left(\frac{5n^2-2}{6n}\right). \tag{4}$$

The first term in (4) corresponds to the expected travel along the picking length, the second term is the expected distance to traverse the width of the cross aisles, and the third term is the expected cross aisle travel. To evaluate the efficiency of dual-command cycles, relative to single-command cycles, we compare $2E[SC]$ to $E[DC]$. While the exact savings depends on the values of L, v, and a, our empirical studies show a range of 16–33%, over a variety of warehouse shapes and sizes, with the maximum savings occurring for very tall warehouses with few aisles.

It can be shown that the expected single-command travel distance, expressed by (1), is equivalent to the models developed by Roodbergen and Vis (2006) for picking tours (when the number of picks equals one). For dual-command travel (two picks) their models produce slightly longer distances than (4) (6–12% longer for their examples), which is to be expected since they model the S-shaped and largest-gap routing heuristics and we model an optimal path.

3.2 Layout B: Picking Aisles Perpendicular to the Front Wall with a Middle Cross Aisle

Layout B, as depicted in Figure 3(a), has a middle cross aisle of width $2v$, half-way between the top and bottom cross aisles. As before, the warehouse has n picking aisles, each with picking length L, however it is slightly larger due to the middle cross aisle. Given a P&D point at the bottom center of the warehouse and an even number of picking aisles, the expected single-command travel distance for Layout B is the expected single-command travel distance for Layout A, plus the expected travel distance across the middle cross aisle, or $2v$ (because the middle cross aisle must be traversed for 50% of the locations). Therefore, from (1) we get

$$E[SC_{\mathrm{B}}] = L + 4v + \frac{an}{2}. \tag{5}$$

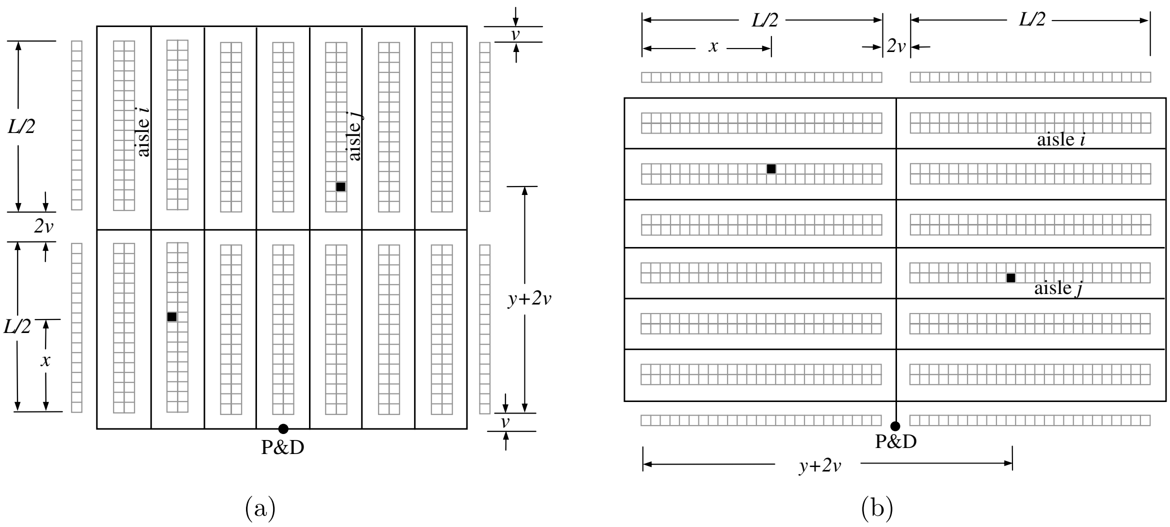


Figure 3: (a) Layout B: Picking Aisles Perpendicular to Front Wall (b) Layout C: Picking Aisles Parallel to Front Wall

As before, to determine the expected travel-between distance for Layout B we consider cross aisle travel and picking aisle travel. The expected cross aisle travel distance is identical to the cross aisle travel in Layout A and described by (2). To determine the expected picking aisle travel, we condition on whether the locations are above or below the middle cross aisle. The expected picking aisle travel distance in this case, which is derived in the appendix, is

$$\frac{1}{n}\left(\frac{L}{3} + v\right) + \frac{n-1}{n}\left(\frac{5}{12}L + 2v\right). \tag{6}$$

The total expected travel distance between two locations for Layout B is then

$$E[TB_{\mathrm{B}}] = \frac{1}{n}\left[\frac{L}{3} + v + (n-1)\left(\frac{5}{12}L + 2v\right)\right] + \frac{a(n^2-1)}{3n}, \tag{7}$$

and the resulting dual-command travel distance for an even number of aisles is

$$E[DC_{\mathrm{B}}] = L\left(\frac{17n-1}{12n}\right) + v\left(\frac{6n-1}{n}\right) + a\left(\frac{5n^2-2}{6n}\right). \tag{8}$$

Comparing $2E[SC]$ to $E[DC]$, the total savings gained by using dual-command travel in Layout B depends on the values of L, v, and a. However, we have seen a range of 19–33% reduction in dual-command travel distance over a variety of warehouse sizes and shapes. These savings are consistently higher than for similarly-sized warehouses of Layout A, which can be attributed to the fact that travel-between is much more efficient in Layout B, therefore dual-command travel is correspondingly more efficient when compared to single-command travel.

3.2.1 Optimal Placement of the Middle Cross Aisle

The middle aisle in our model is equally-spaced between the top and bottom cross aisles, which is consistent with other research (Vaughan and Petersen, 1999; Roodbergen and de Koster, 2001a,b). However, this is not necessarily the optimal position. Roodbergen and de Koster (2001b) propose that a middle aisle closer to the rear of the warehouse (farther from the P&D point) might be better for pick tours, but conclude that the exact middle is close to optimal when picks are uniformly distributed.

Consider a warehouse in which the middle aisle need not be halfway between the top and bottom cross aisles. Let the portion of each picking aisle below the middle cross aisle be αL in length, and the portion of each picking aisle above the middle cross aisle be $(1-\alpha)L$ in length, where $0 \leq \alpha \leq 1$ (the cross aisle is in the center of the warehouse when $\alpha = 0.5$). Cross aisle travel in not affected by the position of the middle cross aisle, therefore we focus on picking aisle travel. Using the method outlined in the appendix, and maintaining the assumption that the locations are uniformly distributed in the warehouse, the expected picking aisle travel distance in the more general case is

$$\frac{1}{n}\left[\frac{L}{3} + 4\alpha(1-\alpha)v\right] + \frac{n-1}{n}\left[\left(\alpha^2 - \alpha + \frac{2}{3}\right)L + 2v\right].$$

With the cross aisle travel expressed by (2), the expected travel-between distance in Layout

B when α is a variable is

$$E[TB_{\mathrm{B}(\alpha)}] = \frac{1}{n}\left[\frac{L}{3} + 4\alpha(1-\alpha)v\right] + \frac{n-1}{n}\left[\left(\alpha^2 - \alpha + \frac{2}{3}\right)L + 2v\right] + \frac{a(n^2-1)}{3n}.$$

This function is minimized when $\alpha = 0.5$. We also consider picking aisle travel in a single-command cycle, where the middle aisle is traversed $(1-\alpha)\%$ of the time. The expected one-way travel distance is $v + L/2 + (1-\alpha)2v$, which leads to a roundtrip of $L + (6-4\alpha)v$. The expected single-command travel distance is then

$$E[SC_{\mathrm{B}(\alpha)}] = L + (6-4\alpha)v + \frac{an}{2},$$

which is minimized when α is at its maximum of 1.0. This result is consistent with the earlier result that adding a middle cross aisle increases travel from the P&D point to the pallet positions above the middle aisle. Because the expected dual-command travel distance is the sum of the expected travel-between distance and the expected single-command travel distance, we have the following proposition.

Proposition 1 *In Layout B, the position of the middle aisle that minimizes dual-command travel distance is between the center of the warehouse and the top cross aisle.*

Proof. Expected single-command travel distance is minimized when the middle aisle is as far above the center as possible. Expected travel-between distance is minimized when the middle aisle is placed exactly in the center. Since the expected dual-command travel distance is a convex combination of these two components, the result is an optimal position that is somewhere between the exact center and the top cross aisle. $\square$

In an empirical illustration of this result, we assume square pallet footprints (which include clearances), and specify the warehouse dimensions in pallets, where the center-to-center distance between adjacent picking aisles, a, is 5 pallets, and the cross aisle width, $2v$, is 3 pallets. Using fixed picking aisle lengths $L = 10$, 20, 50 and 100 pallets, we determine the optimal position of the middle aisle, α^*, for values of n up to $n = 40$. The results are shown in Figure 4. We note that α^* is greater for small values of L, and that for the more common aisle lengths ($L = 50$ and 100), α^* is relatively constant for a given L. The practical application of this result is that when a storage area is expanded to include additional picking aisles of the same length, the optimal position of the cross aisle does not change.

How much do we gain by placing the middle cross aisle in the optimal position? Figure 5 shows the percent improvement in expected dual-command travel distance when the cross aisle is moved from the exact middle to the optimal position, as indicated in Figure 4. Except

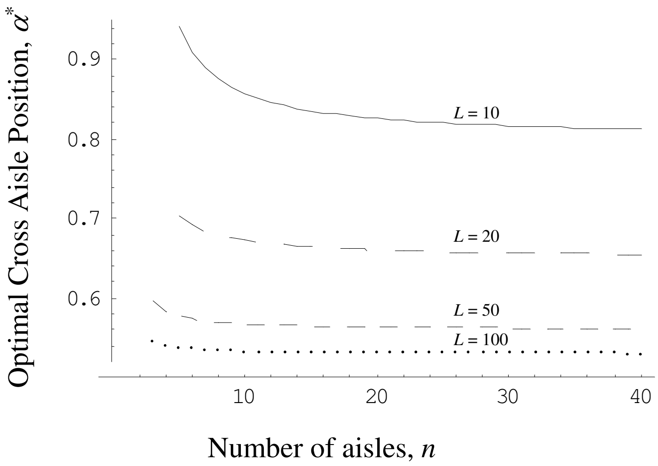


Figure 4: Optimal Cross Aisle Position versus n for $L = 10$, 20, 50 and 100

for very small warehouses, the improvement is less than 1%; therefore we can conclude that a middle cross aisle placed halfway between the top and bottom cross aisles ($\alpha = 0.5$) gives close-to-optimal performance. For this reason, the remaining analysis in this paper with Layout B assumes $\alpha = 0.5$.

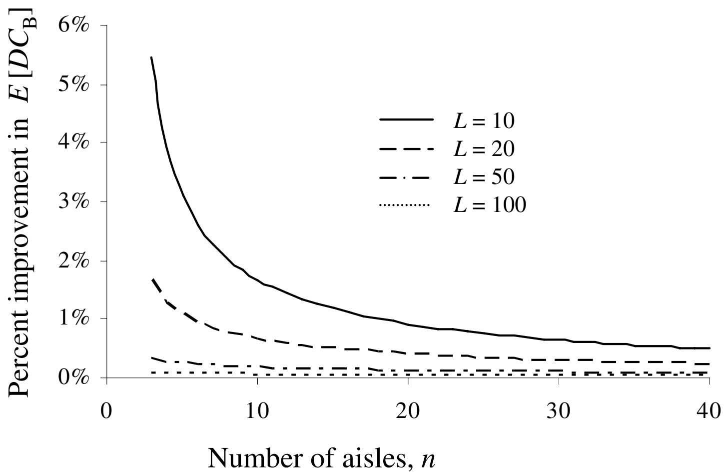


Figure 5: Improvement in $E[DC_{\mathrm{B}}]$ Gained by Using α^* rather than $\alpha = 0.5$, for $L = 10$, 20, 50 and 100

3.3 Layout C: Picking Aisles Parallel to Front Wall with a Central Cross Aisle

Layout C, represented in Figure 3(b), is similar to Layout B except that the picking aisles are parallel to the front wall of the warehouse where the P&D point is located, and a central cross aisle perpendicular to the front wall is inserted. The two layouts can actually be viewed as the same general design with the P&D point moved to another position. As we will see, however, this change in the position of the P&D point has a significant impact on the optimal shape of the warehouse, therefore it is beneficial to view Layouts B and C as distinct designs. As before, the warehouse has n picking aisles, each with picking length L, and cross aisles of width $2v$.

Given a P&D point at the bottom center of the warehouse, as shown in Figure 3(b), the expected single-command travel distance for Layout C is

$$E[SC_{\mathrm{C}}] = \frac{L}{2} + 2v + an. \tag{9}$$

The expected travel between two locations in Layout C is the same as the expected travel-between distance in Layout B, where $E[TB_{\mathrm{C}}]$ is given by (7). The resulting dual-command travel distance for this warehouse is then

$$E[DC_{\mathrm{C}}] = L\left(\frac{11n-1}{12n}\right) + v\left(\frac{2n-1}{n}\right) + a\left(\frac{4n^2-1}{3n}\right). \tag{10}$$

The total savings gained by using dual-command travel in Layout C is similar to the results seen for Layout B. The question of optimal inserted cross aisle position is not an issue for Layout C due to the fact that the center position minimizes both $E[SC_{\mathrm{C}}]$ and $E[TB_{\mathrm{C}}]$, which therefore minimizes $E[DC_{\mathrm{C}}]$.

The next section uses the expressions for single-command and dual-command travel distance in Layouts A, B and C, to determine the warehouse shape that minimizes travel distance.

4 Optimal Warehouse Shape

The models developed by Roodbergen and Vis (2006) for travel distance in a "one block" warehouse (Layout A), allow them to determine the optimal number of aisles, n, for a fixed total picking length and picklist size, by enumerating the possible solutions for n. By relaxing the integrality of n, we can directly calculate the optimal number of aisles for single-command and dual-command operations.

4.1 Single-Command Travel

For single-command travel, if we let T be the total picking length of the warehouse, substituting $L = T/n$ into our three equations for $E[SC]$ and taking the derivative with respect to n, yields $n^*(SC)$, or the number of aisles that minimizes single-command travel distance. We consider Layouts A, B and C, and for ease of comparison, assume Layouts A and B have an even number of aisles. For Layouts A and B,

$$n^*(SC_{\mathrm{A}}) = n^*(SC_{\mathrm{B}}) = \sqrt{\frac{2T}{a}}, \tag{11}$$

and for Layout C,

$$n^*(SC_{\mathrm{C}}) = \sqrt{\frac{T}{2a}}. \tag{12}$$

Using the 2nd-order condition for convexity, we can show that for all three layouts, $\frac{\partial^2 E[SC]}{\partial n^2} > 0$ for all $n > 0$, therefore $E[SC]$ is convex with respect to n, and the optimal number of aisles is achieved by rounding either up or down; i.e., $\min(\lceil n^*(SC) \rceil, \lfloor n^*(SC) \rfloor)$. Characterizing the shapes described by (11) and (12) leads to the following proposition.

Proposition 2 *The warehouse shape that minimizes single-command travel in Layouts A, B and C is approximately half as tall as it is wide.*

Proof. For Layouts A and B, if the cross aisle width, $2v$, is small relative to the picking aisle length, L, then for a warehouse that is half as tall as it is wide, $an/2 \approx L = T/n$; and by rearranging terms, $n = \sqrt{2T/a}$, which is the same as (11). Likewise, for Layout C, the width of the warehouse is approximately $L = T/n$, and the height is an; therefore, $T/2n = an$ and $n = \sqrt{T/2a}$, or the result in (12). $\square$

The above result is shown by Francis (1967) for a continuously-represented warehouse, and stated by Bassan et al. (1980) for Layouts A and C. For a given T and shapes optimized for single-command travel, Layout C has half the number of aisles as Layouts A and B, but they are twice as long. (Recall that we assumed an even number of aisles for Layouts A and B.) Furthermore, when Layouts A and C are optimally-shaped, the expected single-command travel distance is the same. In this scenario, the main difference between the two layouts is access to the picking space from the bottom cross aisle: Layout C has a dominant single P&D point, whereas the picking space can be accessed from anywhere along the bottom aisle. Note that there is little to no difference between the area of the two layouts when Layouts A and C are optimally-shaped.

4.2 Dual-Command Travel

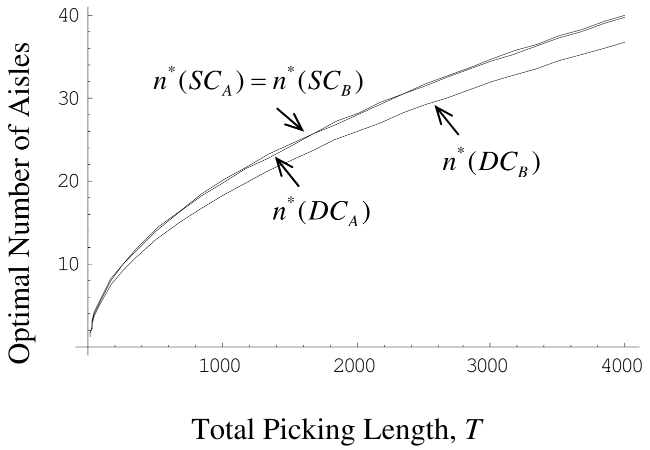
For dual-command travel, we again relax the integrality of n and take the derivative of $E[DC]$ with respect to n. For Layout A, this results in the cubic equation $5an^3 + (5a - 10T + 12v)\,n + 4T = 0$. Solving for the optimal n yields

$$n^*(DC_{\mathrm{A}}) = \frac{-b}{3\left[-\frac{c}{2} + \sqrt{\left(\frac{c}{2}\right)^2 + \left(\frac{b}{3}\right)^3}\right]^{1/3}} + \left[-\frac{c}{2} + \sqrt{\left(\frac{c}{2}\right)^2 + \left(\frac{b}{3}\right)^3}\right]^{1/3}, \qquad (13)$$

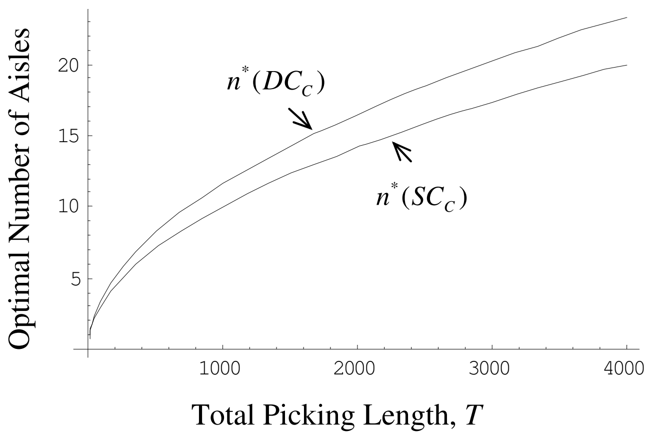
where $b = (5a - 10T + 12v)/5a$ and $c = 4T/5a$. For Layouts B and C, solving for the optimal n in the same manner also yields (13), except that for Layout B, $b = (10a - 17T + 12v)/10a$ and $c = T/5a$, and for Layout C, $b = (4a - 11T + 12v)/16a$ and $c = T/8a$. For reasonable parameter values (where T is at least several times larger than v or a) we can show that $E[DC]$ is convex for all positive integer values of n, therefore an optimal number of aisles can be found by rounding the result of (13) either up or down. Note that evaluating (13) can be difficult to do by hand, because, although it produces real-valued results, the intermediate steps may require manipulation of complex numbers. For this reason, it is simpler to find the roots of the cubic equation using a mathematical software package, such as Mathematica (2005).

Using these results, we examine how the shape of a warehouse that is optimized for dual-command travel distance is different from the one that is optimized for single-command travel distance (which we showed in Section 4.1 is a square half-warehouse for all three layouts). Figures 6(a) and 6(b) show the optimal number of aisles versus total picking length for Layouts A and B, and for Layout C, respectively. As before, we assume all aisles are 3 pallets wide ($2v = 3$ and $a = 5$). For Layout A, we see that the curves for $n^*(SC_{\mathrm{A}})$ and $n^*(DC_{\mathrm{A}})$ are almost the same (although (11) and (13) look very different), therefore the optimal warehouse shape is approximately the same. For Layout B, however, the warehouse optimized for dual-command travel distance has fewer aisles, i.e., it is narrower and taller, than the warehouse optimized for single-command travel. For Layout C, the curve for $n^*(DC_{\mathrm{C}})$ is above the curve for $n^*(SC_{\mathrm{C}})$, therefore the warehouse optimized for dual-command travel has a greater number of aisles, or is narrower and taller, as we saw for Layout B. For our values of v and a, the shape that minimizes dual-command travel distance in both Layouts B and C follows, on average, height/width = 0.65.

What is the impact of designing a warehouse with a non-optimal number of aisles? In Figure 7 the expected dual-command travel distance for Layouts A, B and C is plotted versus



(a)



(b)

Figure 6: Optimal Number of Aisles for Single-Command and Dual-Command Travel: (a) Layouts A and B, (b) Layout C

warehouse shape factor (height/width) for three different warehouse sizes (T = 300, 1000 and 3000). The wrong warehouse shape can increase the dual-command travel distance by more 30%, however, the curves are somewhat flat ($E[DC]$ is within 5% of optimal) when the shape factor is in the range of 0.4 to 1.

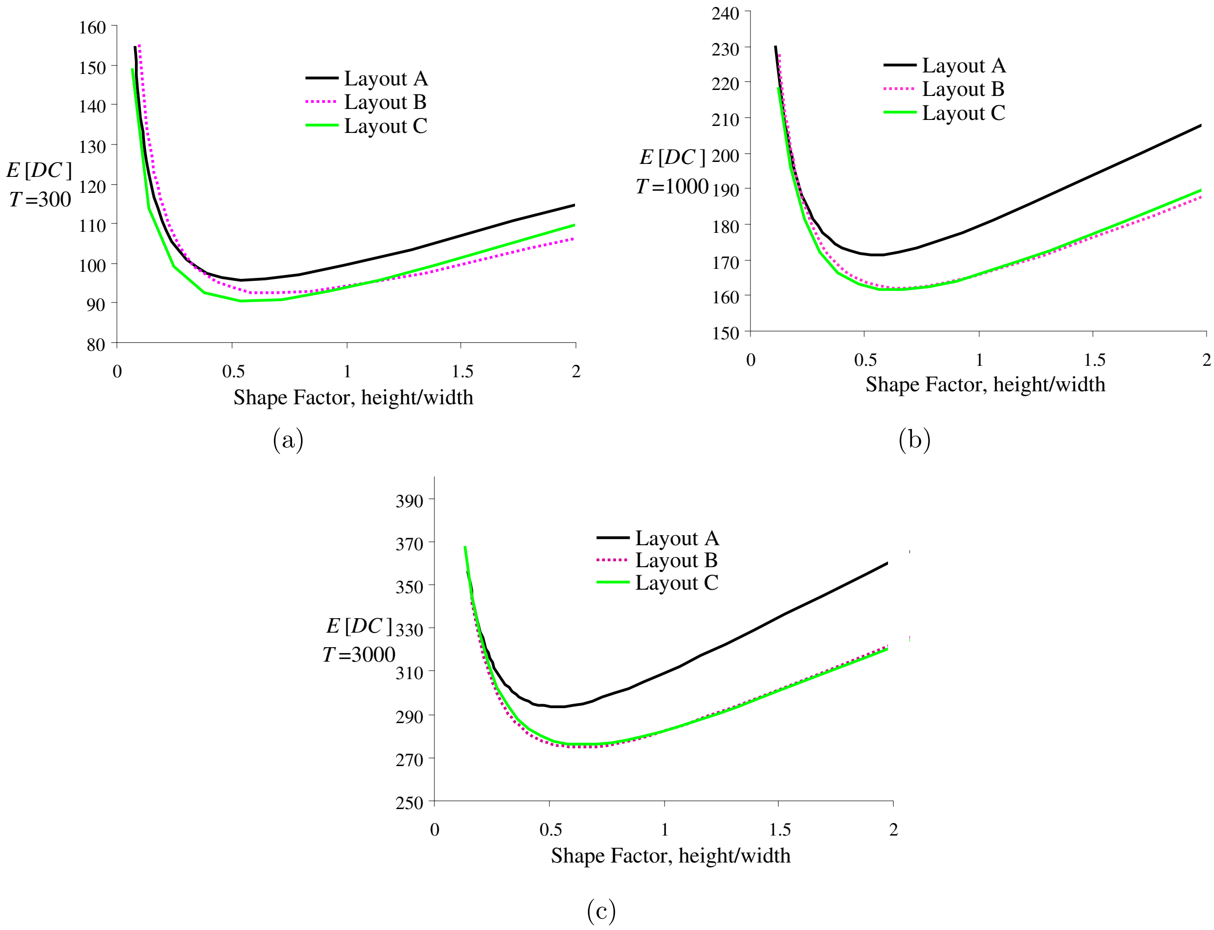


Figure 7: Effect of Warehouse Shape on Dual-Command Travel for (a) $T = 300$ (b) $T = 1000$ and (c) $T = 3000$

5 Performance Results for Layouts A, B and C

In this section we compare the performance of Layouts A, B and C for dual-command operations. We first provide some analytical results that illustrate the benefit of a middle cross aisle for dual-command travel by comparing Layouts A and B, and then Layouts A and C. Finally, we compare all three aisle designs, given that each has been optimized for dual-command travel distance.

5.1 A versus B: When is the Addition of a Middle Cross Aisle Beneficial?

Layout B is essentially the same aisle design as Layout A, with a middle cross aisle inserted. For this configuration, adding a middle cross aisle degrades the "out and back" portion of a tour, but generally improves the travel between pallet locations. This is confirmed in Vaughan and Petersen (1999) and Roodbergen and de Koster (2001a,b) using simulation, where we see reduced expected travel distance for as few as two picks, depending on the warehouse layout. Since there is clearly a tradeoff between how much penalty we are willing to tolerate (width of the cross aisle), to save on picking aisle travel, we propose a rule, based on the length of the picking aisles and the width of the cross aisles, for establishing when a middle cross aisle is useful for dual-command travel.

Proposition 3 *A dual-command warehouse should have a middle cross aisle half-way between the top and bottom cross aisles if*

$$\frac{L}{2v} > \frac{4n+2}{n-1}.$$

Proof. Since the cross aisle travel is the same for both types of warehouses, we focus on the picking aisle travel. In other words, we compare the picking aisle travel distance in Layout B, $L(17n-1)/(12n) + v(6n-1)/n$, with the picking aisle travel distance in Layout A, $L(5n-1)/(3n) + v(4n-2)/n$, to see when the former is less than the latter. Algebraic manipulation gives the result. □

Because only picking aisle travel is considered, Proposition 3 is independent of the P&D location and whether the number of picking aisles is odd or even. Proposition 2 leads to two corollaries, which can both be shown by recognizing that for integer values of $n > 1$, $(4n+2)/(n-1)$ is a strictly decreasing function.

Corollary 1 *The upper bound on* $(4n+2)/(n-1)$ *is* 10*, which occurs at* $n = 2$*. Therefore the addition of a middle cross aisle would improve dual-command travel in any warehouse with* $L/2v \geq 10$*.*

Corollary 2 *The lower bound on* $(4n+2)/(n-1)$ *is* 4*, which occurs as* $n \to \infty$*. Therefore the addition of a middle cross aisle would degrade dual-command travel in any warehouse with* $L/2v \leq 4$*.*

5.2 A versus C: Does Layout C Dominate Layout A?

Due to the inserted cross aisle, dual-command travel in Layout C is, in more likely to be more efficient than in Layout A. In fact, we show with the following proposition that for warehouses of similar shape, Layout C is more efficient than Layout A for dual-command travel.

Proposition 4 *Given a total picking length T, and warehouses that are approximately half as tall as they are wide; i.e., the number of aisles is given by* (11) *and* (12)*, expected dual-command travel distance is less in Layout C than in Layout A.*

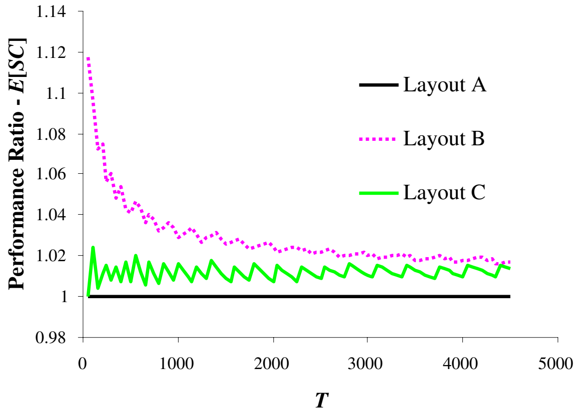
Proof. When the number of aisles in Layouts A and C are given by (11) and (12), respectively, then $n_{\mathrm{A}} = 2n_{\mathrm{C}}$. Substitution into (4) and (10), with algebraic manipulation results in $E[DC_{\mathrm{C}}] < E[DC_{\mathrm{A}}]$ for all positive values of T, v and a. □

We illustrated empirically in Section 4.2 that the shape that minimizes dual-command travel in Layout A is approximately half as tall as wide, but the shape that minimizes dual-command travel in Layout C is narrower and taller. Therefore, the comparison in Proposition 4 is a comparison of an optimal Layout A to a sub-optimal Layout C. We can therefore conclude that when Layouts A and C have both been optimized in shape for dual-command travel, expected dual-command travel distance is always less in Layout C. This statement is illustrated by the numerical results in Section 5.3.

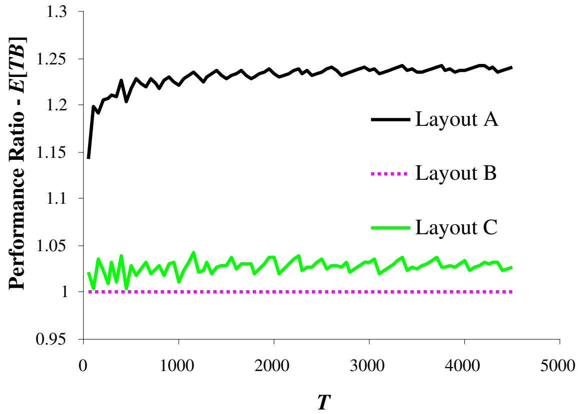
5.3 Numerical Results

In this section we compare the performance of Layouts A, B and C, each designed to minimize dual-command travel distance. Figure 8 shows $E[SC]$, $E[TB]$ and $E[DC]$ over a range of values of T, where the warehouses evaluated each have the number of aisles that minimizes $E[DC]$. We plot the performance ratio of each design, which is defined for $E[SC_{\mathrm{A}}]$ as $E[SC_{\mathrm{A}}]/\min(E[SC_{\mathrm{A}}], E[SC_{\mathrm{B}}], E[SC_{\mathrm{C}}])$. Note in Figure 8(a) that Layout A is preferable for single-command travel, and in Figure 8(b) that Layout B is most efficient for travel-between.

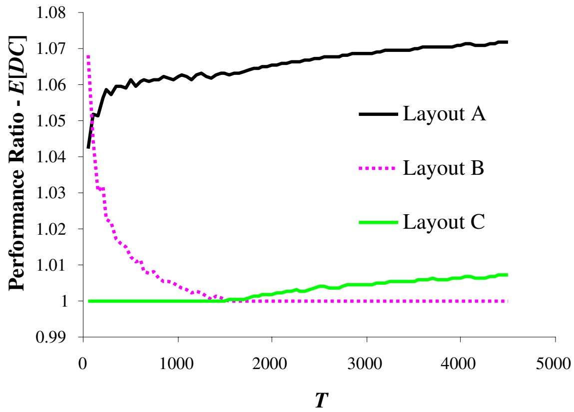
The resulting difference for dual-command travel is shown in Figure 8(c), where Layouts B and C are approximately 5% more efficient than Layout A. Layout C is preferred for small warehouses ($T < 1500$), while Layout B is preferred for larger warehouses ($T > 1500$). The difference between Layout B and C is small, being within 1% of each other for values of $T > 500$.



(a)



(b)



(c)

Figure 8: Performance Ratio (PR) Comparisons for Layouts A, B and C: (a) Single-Command Travel (b) Travel-Between and (c) Dual-Command Travel

6 Conclusions

Dual-command operations are common in the warehousing industry today, and, with the increasing use of warehouse management systems, we expect the practice to increase in popularity. We should note that even in warehouses that use dual-commands, single-command operations are still common, due to unavoidable and sometimes purposeful imbalances in receiving and shipping workloads.

For manual storage areas such as we address in this paper, the academic literature has focused almost exclusively on designing for single-command operations. Our work is intended to lend some insight into designs for dual-command operations.

We considered three common layouts for unit-load warehouses in the context of dual-command operations. The first (Layout A) is a very common design that has two cross aisles at the ends of the picking aisles. The other two designs (Layouts B and C) insert a middle cross aisle to facilitate the travel between locations. We developed analytical expressions for expected dual-command travel distance in all three layouts and used them to determine when it is best to have a middle cross aisle or not, and when each layout might be preferred.

We showed that one of the two designs with an inserted cross aisle (Layout C) always outperforms the design without the inserted cross aisle (Layout A) for dual-command operations. Furthermore, Layout B outperforms Layout A for all but very small warehouses. Most interesting to us was that Layout C dominated Layout B for a wide-range of parameters and when it did not, it was within 1 percent of Layout B. These results should be of interest to the industry because—in our experience—Layout B is much more common in practice than Layout C. This could be because Layout C is more dependent on the assumption of one, central P&D location. Finally, we showed that the optimal placement of the "middle cross aisle" of Layout B is not in the middle, but above it.

The research could be extended in several ways. First, warehouses with more than one inserted cross aisle could be modeled, although to do so analytically may be difficult. Second, our results are based on an assumption of one, central P&D location; models that relax this assumption would be welcome. Third, our results are based on the assumption of random storage, but since turnover-based storage policies are another method of improving warehouse performance, models for such storage policies would be useful. And finally, we assume storage and retrieval requests are processed in a first-come, first-served manner, but even with dual-command operations, there exists the potential to decrease the cycle time by opportunistically pairing requests.

Appendix

Expected Travel-Between Distance for Layout A. To determine the expected travel distance between two random locations, we divide the travel-between distance into two components: (1) cross aisle travel, and (2) picking aisle travel — including the distance v to enter/exit the picking aisles, if necessary. We assume the locations are uniformly distributed among the aisles. Since the aisles are equivalent in length, all aisles are equally likely to contain one of the two locations. If the number of aisles is n, there are n^2 possible combinations of i and j, all equally likely. For instance, there are n possible ways that $i = j$, for which there is no cross aisle travel required. Therefore, the probability that $|i - j| = 0$ is n/n^2. Likewise, there are $2(n-1)$ ways that $|i - j| = 1$, so $Pr(|i - j| = 1) = 2(n-1)/n^2$.

The expected number of aisle widths between i and j is

$$E[|i-j|] = \sum_{k=0}^{n-1} k\, Pr(|i-j| = k) = \sum_{k=1}^{n-1} k\, \frac{2(n-k)}{n^2} = \frac{(n^2-1)}{3n}.$$

The expected cross aisle travel distance is then

$$\frac{a(n^2-1)}{3n}. \tag{2}$$

For picking aisle travel, we consider the case where the locations are in the same picking aisle, and the case where the locations are in different picking aisles. Since we assume a continuous uniform distribution within each aisle, we let X_i and Y_j be uniform random variables that represent the position of the locations in aisles i and j, respectively, where $X_i \sim U(0, L)$ and $Y_j \sim U(0, L)$. From probability theory, we know the expected distance between two locations on the same aisle of length L is $E[|x-y|] = L/3$. For locations that are on different picking aisles, the picking aisle travel distance is $\min[x+y, 2L-(x+y)]+2v$. We let $Z_{ij} = X_i + Y_j$. The convolution of two identical uniform density functions is triangular. Using the probability density function of Z_{ij}, $f_Z(z)$, we find that the expected value of $\min[x+y, 2L-(x+y)]$ is then

$$E[\min(z, 2L-z)] = \int_0^{2L} \min(z, 2L-z) f_Z(z) dz = \int_0^{L} z f_Z(z) dz + \int_L^{2L} (2L-z) f_Z(z) dz = \frac{2}{3}L.$$

The probability the second location will be in the same aisle as the first location is $1/n$, and the probability the second location will be in a different aisle is $(n-1)/n$. The expected picking aisle travel distance is then

$$\frac{1}{n}\left(\frac{L}{3}\right) + \frac{n-1}{n}\left(\frac{2}{3}L + 2v\right). \tag{3}$$

Expected Travel-Between Distance for Layout B. For Layout B, the expected cross aisle travel is identical to the cross aisle travel in Layout A, as described by (2). For picking aisle travel, X_i and Y_j are still random variables, such that $X_i \sim U(0, L)$ and $Y_j \sim U(0, L)$, however, travel across the middle cross aisle must be considered. In Figure 3(a), the left pallet location is a distance of x above the bottom of picking aisle i, but the right location is a distance of $y + 2v$ above the bottom of picking aisle j. When the two locations are in the same picking aisle, we expect to traverse the middle cross aisle half the time, therefore, the expected travel is $L/3 + 2v(1/2) = L/3 + v$. To evaluate picking aisle travel distance when the locations are in different aisles, we condition on whether the locations are above or below the middle cross aisle and consider four mutually exclusive cases:

$$(1)\ x \leq \frac{L}{2}, y \leq \frac{L}{2} \qquad (2)\ x > \frac{L}{2}, y > \frac{L}{2} \qquad (3)\ x \leq \frac{L}{2}, y > \frac{L}{2} \qquad (4)\ x > \frac{L}{2}, y \leq \frac{L}{2}.$$

In case 1, both locations are below the middle cross aisle, therefore the shortest path uses either the bottom cross aisle or the middle cross aisle. The picking aisle travel is this case is $\min[x+y, L-(x+y)]+2v$. Applying the results from Layout A (with an aisle length of $L/2$) yields expected picking aisle travel distance of $L/3+2v$ for case 1. In case 2, both locations are above the middle cross aisle, and by similarity with case 1, the expected picking aisle travel distance is also $L/3+2v$.

In cases 3 and 4, we have one location above the middle cross aisle and one location below. In these cases it is always optimal to use the middle cross aisle, therefore the expected picking aisle travel distance is $|x-y|+2v$. For case 3,

$$E\left[Y - X|X \leq \frac{L}{2}, Y > \frac{L}{2}\right] = E\left[Y|Y > \frac{L}{2}\right] - E\left[X|X \leq \frac{L}{2}\right] = \frac{3L}{4} - \frac{L}{4} = \frac{L}{2},$$

therefore the expected picking aisle travel distance is $L/2+2v$. By similarity, this applies for case 4 as well. Since cases 1–4 are equally likely, the expected picking aisle travel distance when the locations are on different aisles is

$$\left[\frac{L}{3}\left(\frac{1}{4}\right) + \frac{L}{3}\left(\frac{1}{4}\right) + \frac{L}{2}\left(\frac{1}{4}\right) + \frac{L}{2}\left(\frac{1}{4}\right)\right] + 2v = \frac{5}{12}L + 2v.$$

The expected picking aisle travel distance is then

$$\frac{1}{n}\left(\frac{L}{3}+v\right) + \frac{n-1}{n}\left(\frac{5}{12}L+2v\right). \tag{6}$$

Acknowledgements

This research was supported in part by the National Science Foundation under Grants DMI-0600374 and DMI-0600671.

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