Optimizing Fishbone Aisles for Dual-Command Operations in a Warehouse

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Abstract

Unit-load warehouses store and retrieve unit-loads, typically pallets. When storage and retrieval operations are not coordinated, travel is from a pickup and deposit (P&D) point to a pallet location and back again. In some facilities, workers interleave storage and retrieval operations to form a dual-command cycle. Two new aisle designs proposed by Gue and Meller (2006) use diagonal aisles to reduce the travel distance to a single pallet location by approximately 10 and 20 percent for the two designs, respectively. We develop analytical expressions for travel between pallet locations for one of these — the fishbone design. We then compare fishbone warehouses that have been optimized for dual-command to traditional warehouses that have been optimized in the same manner, and show that an optimal fishbone design reduces dual-command travel by 10–15 percent.

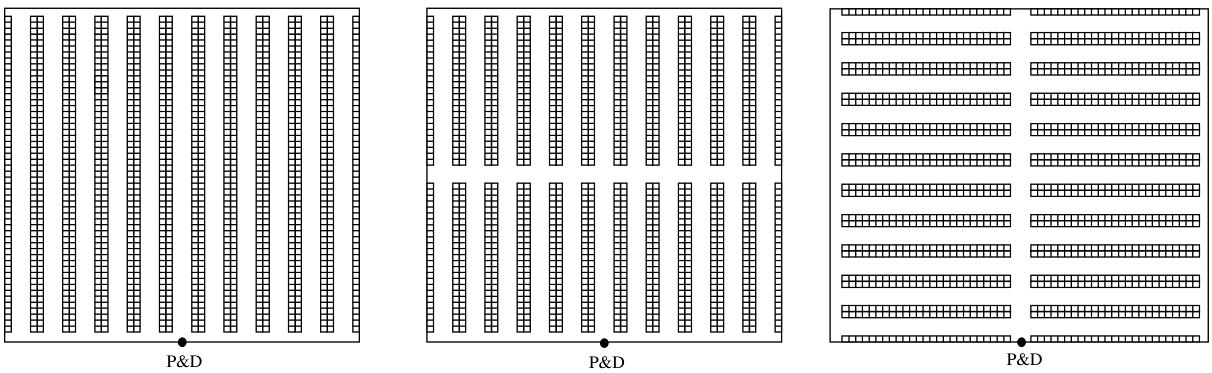
1 Introduction

A unit-load warehouse receives and ships material in single discrete units, usually pallet loads. Unit-load warehouses are commonly found in industry, and include third-party transshipment warehouses, beverage distributors and import distribution centers. Many warehouses that ship in smaller units, such as cartons or pieces, have a portion of their activity dedicated to unit-loads. For example, the reserve area of a distribution center may be used to receive and store pallets until they are needed to replenish a fast-pick area, when they are then "shipped" to another part of the warehouse (Bartholdi and Hackman, 2007). As discussed in Pohl, Meller and Gue (2008), import warehouses play a critical role in today's global economy, which suggests a renewed investigation into unit-load warehouse design.

Storage and retrieval operations are labor-intensive, with the majority of workers' time spent in travel. Because modern warehouses are larger than ever before, travel is an even more significant factor. According to Hudgins (2006): "A decade ago, 300,000 square feet was the definition of a large warehouse — today it's upwards of 1 million square feet." If we are able to reduce the required travel distances, we can see improvements in two ways: either by reducing labor costs, or, what may be more important in a competitive market, by achieving a faster response time to provide better customer service.

One method of reducing travel in a unit-load warehouse is to coordinate, or interleave, storage and retrieval operations. Travel from a common pickup and deposit (P&D) point to a single pallet location and back again is referred to as a single-command cycle. In a dual-command cycle, the worker performs a storage operation and then moves directly to a retrieval location before returning to the P&D point. In a single-command cycle, the forklift is empty for half of the total travel distance, while in a dual-command cycle the empty travel is only about one third of the total travel distance. We refer to this travel between the storage and retrieval locations as "travel-between." Because dual-command operations make more efficient use of time and resources, and contemporary unit-load warehouses use warehouse management systems to schedule activities, the use of dual-command operations is likely to increase.

Figure 1 shows three traditional warehouse designs, with parallel picking aisles and orthogonal cross aisles at each end of the picking aisles. The layouts in Figures 1(b) and 1(c) each have an additional cross aisle that divides the picking space into two sections. There is a penalty to pay for inserting a middle cross aisle: it reduces floor space that could otherwise be allocated to storage, and therefore requires a slightly larger facility to maintain the same amount of storage. However, when more than one location is visited in a single tour, the addition of a cross aisle creates more possible routes for travel between locations and therefore has the potential to reduce expected travel (Roodbergen and de Koster, 2001). For single-command travel, Layouts A and C are preferred, while Layouts B and C are generally preferred for dual-command travel (Pohl, Meller and Gue, 2008). In fact, Pohl, Meller and Gue show that when the number of aisles in Layouts B and C is chosen to minimize expected dual-command travel, their performance is very similar and both layouts require the same amount of aisle space.



(a) Layout A (b) Layout B (c) Layout C

Figure 1: Traditional warehouses.

Gue and Meller (2006) point out that traditional designs, such as those in Figure 1, appear to be subject to the unspoken constraints that picking aisles must be parallel to one another, and cross aisles must be perpendicular to the picking aisles. When they relax these constraints, they show that non-traditional aisle layouts can reduce the expected travel distance to a single pallet location. This observation was also used by White (1972) to motivate the design of "radial aisles" in continuously-represented, non-rectangular warehouses. Gue and Meller (2006) proposed two new designs for a unit-load warehouse, where only single-command cycles are considered. These two new designs, which are presented in Figures 2(a) and 2(b), reduce single-command travel distance by about 10% and 20%, respectively, when compared to Layout A (Gue and Meller, 2008).

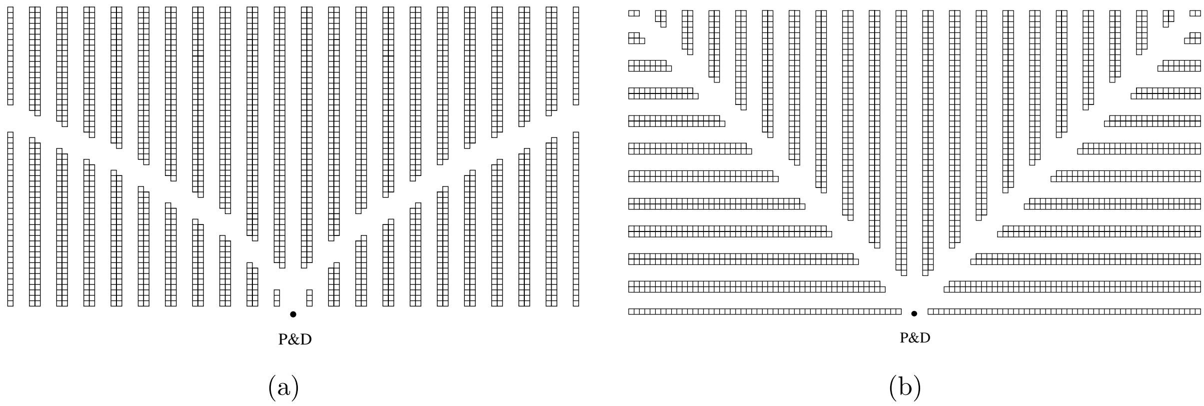


Figure 2: (a) Flying-V warehouse optimized for single-command operations; (b) Fishbone warehouse optimized for single-command operations.

The purpose of this paper is to investigate the fishbone aisle design for dual-command operations. We show how to design a dual-command warehouse with fishbone aisles; and by providing a detailed comparison with traditional aisle designs, we show that the fishbone design provides lower expected travel distances. Our focus is on the use of analytical models of travel distance to optimize warehouse design. The expressions developed here can be used to develop a new fishbone warehouse design, or to modify the aisle layout in an existing facility.

The next section reviews previous research. In Section 3, we develop expressions for travel-between distance in the fishbone aisle design. The travel-between expression is then combined with the single-command distance expression from Gue and Meller (2008) to form a dual-command travel distance expression for any fishbone warehouse. In Section 4, using the new models for dual-command travel, we design optimal fishbone warehouses over a range of sizes, and in Section 5, we compare their performance to equivalent traditional warehouses that are also optimized for dual-command operations. We summarize our results and suggest future work in Section 6.

2 Previous Research

Warehouse design is complex because there are many interrelated design problems that lead to a multitude of potential designs. For this reason, most research focuses on one aspect of the design. Two recent surveys of the literature on warehouse operation and design include Gu et al. (2007) and de Koster et al. (2007). Most of the papers reviewed by these surveys fall into one of two categories: they either consider methods for allocating items to storage locations, or seek to minimize the operational cost of retrieving items from storage through zoning, batching, and/or orderpicker routing. The most common measure of operational cost is expected travel time or travel distance. Several papers develop travel models for the purpose of determining the aisle layout (number and length of aisles, and addition of cross aisles) that minimizes expected travel. Since this paper also develops travel distance models to compare warehouse aisle designs, we focus on the research that models expected travel.

Most of the recent research is concerned with orderpicking. In work that addresses unit-load warehouses, researchers have modeled single-command travel in Layout A (Francis, 1967; Bassan et al., 1980) and Layout C (Bassan et al., 1980), and have presented some well-known results on optimal warehouse shape and P&D location. There are several papers that model dual-command travel in Layout A such as Mayer (1961) and Malmborg and Krishnakumar (1987, 1990), but they do not use their results to optimize warehouse design parameters, such as number and length of aisles. Pohl, Meller and Gue (2008) develop analytical expressions for single-command and dual-command travel in Layouts A, B and C. They derive equations to determine the number of aisles that minimizes dual-command travel in each layout. We use these travel models to describe the traditional warehouse performance that is compared to the performance of fishbone warehouses.

Algorithms to generate optimal orderpicking tours for a given set of pick locations are available (see Ratliff and Rosenthal (1983) and Roodbergen and

de Koster (2001)), however simulation is required to find the average tour length. Authors who develop analytical travel models for orderpicking base them on routing heuristics, rather than optimal tour generation. Because our research is limited to single-command and dual-command travel, our analytical models describe optimal travel paths.

Roodbergen and Vis (2006) and Roodbergen et al. (2008) concentrate, as we do, on finding the best aisle layout given a constant storage capacity. They develop analytical equations for expected travel to a general number of pick locations, assuming routing heuristics. Roodbergen and Vis (2006) consider only a one-block warehouse (Layout A), while Roodbergen et al. (2008) allow a general number of cross aisles and an alternative depot location. Both papers validate their models with simulation.

All of the literature cited above focuses on one or more of the traditional aisle layouts of Figure 1. The nontraditional aisle designs in Gue and Meller (2008) were developed to improve single-command travel in a unit-load warehouse. The first design has a flying-V cross aisle (see Figure 2(a)). The position and orientation of the middle cross aisle is determined by minimizing the expected travel distance from the P&D point, located at the lower center, to a single pallet location. Improvement in single-command travel distance is shown to be approximately 10% for typical warehouse sizes when compared to a traditional warehouse without a middle cross aisle.

The fishbone design, shown in Figure 2(b), has a middle cross aisle that is diagonal and straight, with vertical picking aisles above and horizontal picking aisles below. The slope of the middle cross aisle is determined by minimizing the expected distance from the P&D point, located at the bottom center, to a single pallet location. As mentioned in the previous section, the fishbone design reduces single-command travel by approximately 20% when compared to an equivalent warehouse without a middle cross aisle (Gue and Meller, 2008).

In this paper, we investigate only the fishbone design for dual-command operations, rather than both the fishbone and flying-V designs, because a detailed simulation study (Pohl, Meller and Gue, 2007) showed fishbone to be the most promising of the two new designs for dual-command operations. **As in Gue and Meller (2008), we assume a single P&D point, optimally located in the center of one side of the warehouse (Francis, 1967; Roodbergen and Vis, 2006), and we assume a random storage policy. We therefore model the warehouse as a set of discrete picking aisles, with continuous picking activity uniformly distributed within in each aisle, and among all aisles.** To design fishbone warehouses for dual-command operations, we require an analytical expression for dual-command travel distance. Expressions for single-command travel distance are found in Gue and Meller (2008); therefore, the requirement is for an expression for travel-between distance, which we develop in Section 3.

3 Expected Travel Distance Between Two Locations in a Fishbone Warehouse

We are interested in expected horizontal travel distance. Therefore we do not model the acceleration and deceleration of the vehicle, time to load and unload, or vertical travel to upper pallet positions. Workers can travel in either direction in an aisle and can change directions within an aisle. We assume that the picking aisles are sized such that the racks on each side of the picking aisle can be accessed, but that the lateral travel within the aisle is negligible. That is, we assume the travel distance to a given pallet position is the same as the travel distance to a pallet position directly above or directly across the picking aisle. A "location" in the warehouse therefore refers to two columns of pallet positions, one on each side of the picking aisle. For example, the warehouse shown in Gue and Meller (2008) with 21 picking aisles that are 50 pallets long, would have a total picking aisle length $T = 1050$. This implies that there are 2100 pallet positions on each level of the storage racks. The storage and retrieval requests are assumed to be independent, and processed on a first-come-first-serve basis.

In a fishbone warehouse, there are several potential paths between any two locations, and the optimal path is very much dependent on which two locations in the warehouse are considered. The cross aisles in the fishbone design divide the warehouse into three regions, which are indicated in Figure 3. We approach the problem incrementally by considering the cases for which

1. **travel is within one region (Section 3.1),**

2. **travel is between Regions 1 and 2 (Section 3.2), and**

3. **travel is between Regions 1 and 3 (Section 3.3).**

Due to the symmetry of the warehouse, the expected travel between Regions 2 and 3 is the same as between Regions 1 and 2. The total expected travel-between distance, $E[TB]$, is the weighted sum of the conditional expectations derived in Sections 3.1, 3.2 and 3.3:

$$\begin{aligned} E[TB] &= E[TB^{11}]w_{11} + E[TB^{22}]w_{22} + E[TB^{33}]w_{33} \\ &\quad + 2E[TB^{12}]w_{12} + 2E[TB^{13}]w_{13} + 2E[TB^{23}]w_{23}, \end{aligned}$$

where $E[TB^{rs}]$ is the expected travel between a location in Region r and a location in Region s, and the weights, w_{rs}, are proportional to the product of the total picking aisle lengths in Regions r and s.

3.1 Travel Within a Region

Because the fishbone design is symmetric, Regions 1 and 3 are the same size and shape; therefore, the expected distance between two locations in Region 1 is equal to the expected distance between two locations in Region 3. Region 2 is shaped differently; however it resembles the other two regions, in that it is bordered by a cross aisle on one side that is perpendicular to the picking aisles (the top cross aisle), and a cross aisle on the other side that is diagonal to the picking aisles. Due to this similarity, we can develop a single set of

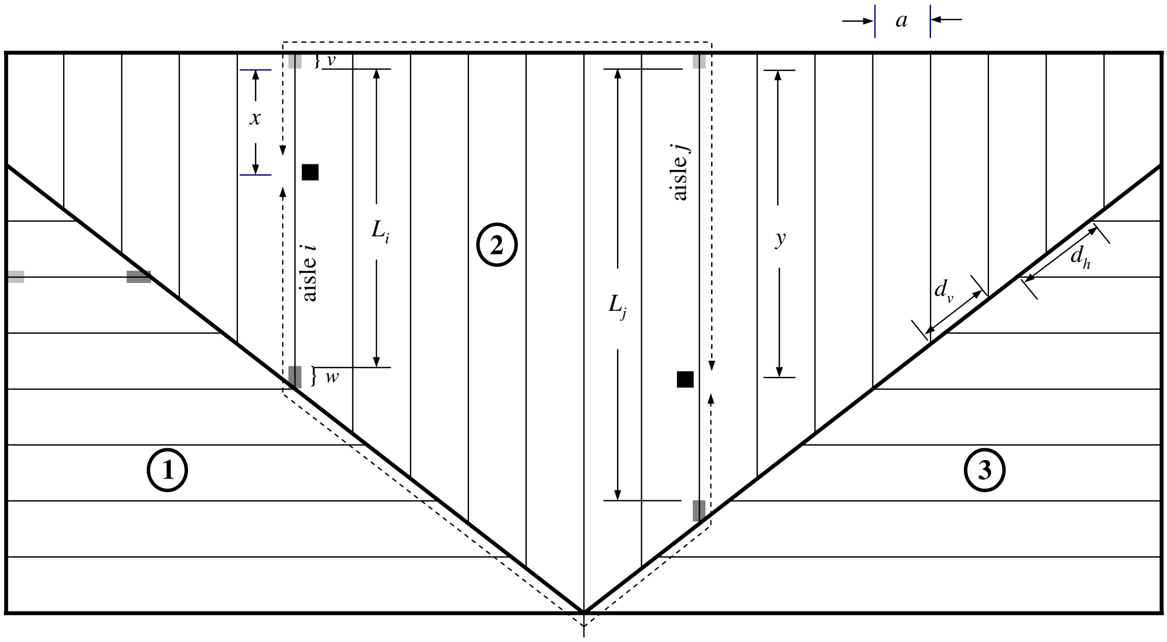


Figure 3: Fishbone regions, with travel paths between two locations in Region 2.

equations to describe travel within any region in the fishbone warehouse. For most of this section we assume the two pallet locations to be visited are in Region 2. With only a slight modification, the results for Region 2 can be extended to describe travel within Regions 1 and 3.

If two locations are in the same picking aisle, no cross aisle should be used for travel-between. If the locations are in different aisles within the region, the travel-between can use either the perpendicular cross aisle or the diagonal cross aisle, as indicated for Region 2 by the dashed lines in Figure 3. The choice between these two alternative paths depends upon the exact locations. The shaded regions in Figure 3 indicate where no picking takes place due to the width of the cross aisles. We assume the perpendicular cross aisle consumes v of each picking aisle, and the diagonal cross aisle consumes w of each picking aisle, where $w \approx \sqrt{2}\, v$.

We define the position of any location by its distance from the start of the aisle, which is the end where the picking aisle intersects the perpendicular cross aisle. Let x denote the distance from the start of aisle i to one location, and y denote the distance from the start of aisle j to the other location. Aisle i has picking length L_i and aisle j has picking length L_j. The picking aisles in each region are numbered sequentially, such that the distance along the perpendicular cross aisle between aisles i and j is $a|i-j|$, where a is the distance between adjacent picking aisles. If the perpendicular cross aisle is used, the distance between locations is $x + v + a|i-j| + v + y$ (refer to Figure 3). For the horizontal picking aisles of Regions 1 and 3, the diagonal distance between aisles is $d_h = a\sqrt{1 + 1/m^2}$, where m is the slope of the diagonal cross aisle. The diagonal distance between the vertical picking aisles of Region 2 is $d_v = a\sqrt{1+m^2}$. If travel is along the diagonal cross aisle, the distance between locations in Region 2 is then $(L_i - x) + w + d_v|i-j| + w + (L_j - y)$.

For most picking aisle pairs i and j, the choice of cross aisle depends on the values of x and y (we analyze this in Section 3.1.1). For other instances of i and j, the shortest path always uses the perpendicular cross aisle, and these cases are discussed in Section 3.1.2. Section 3.1.3 presents the total expected distance for travel between two locations in the same region.

3.1.1 Two Alternative Paths

For most instances of i and j, the choice of cross aisle depends on the values of x and y, or more specifically, the value of $x+y$. The shortest distance between locations is the minimum of the two alternative paths, $x + y + a|i-j| + 2v$ and $L_i + L_j - (x+y) + d_v|i-j| + 2w$. If $x + y$ is small, then the perpendicular cross aisle is the best choice. If $x + y$ is large, the diagonal cross aisle is best. We are indifferent about which path to choose when the two paths are equal in length; i.e., when

$$x + y + a|i-j| + 2v = L_i + L_j - (x+y) + d_v|i-j| + 2w,$$

we see that

$$x + y = \frac{1}{2}\left[L_i + L_j + (d_v - a)|i-j|\right] + w - v.$$

Let q_{ij} be a parameter associated with aisles i and j, that represents the point of indifference in the cross aisle choice. That is,

$$q_{ij} = \frac{1}{2}\left[L_i + L_j + (d_v - a)|i - j|\right] + w - v. \tag{1}$$

We can now compare the value of the variable $x + y$ to the parameter q_{ij}. If $x + y = q_{ij}$ the two alternative paths are the same length (by definition). If $x + y < q_{ij}$, the shortest path between locations uses the perpendicular cross aisle. If $x + y > q_{ij}$, then the shortest path uses the diagonal cross aisle.

Let X_i and Y_j be uniform random variables that represent the position of the locations in aisles i and j, respectively, where $X_i \sim U(0, L_i)$ and $Y_j \sim U(0, L_j)$. We can find the probability density of $Z_{ij} = X_i + Y_j$, which we denote as f_Z, using convolution. The probability that the shortest path between two random locations in aisles i and j is along the *perpendicular* cross aisle is then

$$Pr(Z_{ij} \leq q_{ij}) = \int_0^{q_{ij}} f_Z(z)\, dz,$$

and the probability the shortest path between two random locations is along the *diagonal* cross aisle is

$$Pr(Z_{ij} > q_{ij}) = 1 - Pr(Z_{ij} \leq q_{ij}).$$

The expected value of the travel-between distance, given that the locations are both in Region 2, but in different picking aisles i and j, is

$$\begin{aligned}&(E[Z_{ij}|Z_{ij} \leq q_{ij}] + a|i - j| + 2v)\, Pr(Z_{ij} \leq q_{ij}) \\ &\qquad + (L_i + L_j - E[Z_{ij}|Z_{ij} > q_{ij}] + d_v|i - j| + 2w)\, Pr(Z_{ij} > q_{ij}),\end{aligned}$$

where

$$E[Z_{ij}|Z_{ij} \leq q_{ij}] = \int_0^{q_{ij}} z f_{Z|Z \leq q_{ij}}(z)\, dz \tag{2}$$

and

$$E[Z_{ij}|Z_{ij} > q_{ij}] = \int_{q_{ij}}^{L_i+L_j} z f_{Z|Z>q_{ij}}(z)\, dz. \tag{3}$$

Equations for f_Z, $f_{Z|Z\leq q_{ij}}$ and $f_{Z|Z>q_{ij}}$ are derived in Appendix A. Since f_Z is a piecewise-continuous function, we compute $Pr(Z_{ij} \leq q_{ij})$ for the three intervals of q_{ij}:

$$Pr(Z_{ij} \leq q_{ij}) = F_Z(q_{ij}) = \begin{cases} \dfrac{q_{ij}^2}{2L_iL_j} & \text{for } 0 < q_{ij} < L_i, \\[2ex] \dfrac{q_{ij} - L_i/2}{L_j} & \text{for } L_i \leq q_{ij} < L_j, \\[2ex] \dfrac{2L_iL_j - (q_{ij} - L_i - L_j)^2}{2L_iL_j} & \text{for } L_j \leq q_{ij} < L_i + L_j. \end{cases}$$

Using (2) and (3), and the densities derived in Appendix A, we obtain:

$$E[Z_{ij}|Z_{ij} \leq q_{ij}] = \begin{cases} \dfrac{2}{3}q_{ij} & \text{for } 0 < q_{ij} < L_i, \\[2ex] \dfrac{q_{ij}^2 - L_i^2/3}{2q_{ij} - L_i} & \text{for } L_i \leq q_{ij} < L_j, \\[2ex] \dfrac{(L_i + L_j)q_{ij}^2 - (L_i^3 + L_j^3 + 2q_{ij}^3)/3}{2L_iL_j - (q_{ij} - L_i - L_j)^2} & \text{for } L_i \leq q_{ij} < L_j, \end{cases}$$

and

$$E[Z_{ij}|Z_{ij} > q_{ij}] = \begin{cases} \dfrac{L_i^2L_j + L_iL_j^2 - 2/3q_{ij}^3}{2L_iL_j - q_{ij}} & \text{for } 0 < q_{ij} < L_i, \\[2ex] \dfrac{L_i^2/3 + L_iL_j + L_j^2 - q_{ij}^2}{2(L_i/2 + L_j - q_{ij})} & \text{for } L_i \leq q_{ij} < L_j, \\[2ex] \dfrac{1/3(L_i + L_j)^3 + 2/3q_{ij}^3 - (L_i + L_j)q_{ij}^2}{(q_{ij} - L_i - L_j)^2} & \text{for } L_i \leq q_{ij} < L_j. \end{cases}$$

3.1.2 Using the Perpendicular Cross Aisle: A Special Case

As discussed in the previous section, we generally must know the values of x and y to determine which cross aisle provides the shortest path between two locations. We now address a special case, where the perpendicular cross aisle is always the best choice, and is therefore independent of x and y. Consider, for example, the two shortest picking aisles in Region 2 of Figure 3 (one aisle is on the far left and the other is on the far right). For all values of x and y in those two aisles, the distance along the perpendicular cross aisle is shorter than the distance along the diagonal cross aisle. That is,

$$x+v+a|i-j|+v+y < (L_i-x)+w+d_v|i-j|+w+(L_j-y) \quad \forall\, x \in [0, L_i],\ y \in [0, L_j]. \quad (4)$$

If the inequality in (4) applies for $x = L_i$ and $y = L_j$ (the points furthest from the perpendicular cross aisle), then it applies for all x and y. That is,

$$L_i + L_j < (d_v - a)|i - j| + 2w - 2v, \quad (5)$$

which is independent of x and y. If (5) holds, the perpendicular cross aisle will always provide the shortest path, and the expected value of the distance between the two locations is $(L_i + L_j)/2 + a|i - j| + 2v$. We use this result, along with the results from Section 3.1.1, in the next section to determine the total travel-between distance within a region.

3.1.3 Total Expected Travel Distance Between Locations Within a Region

If both locations are in Region 2, the expected distance between a location in aisle i and a location in aisle j, where i and j are different, is

$$E[TB^{22}_{ij}|i \neq j] = \begin{cases} \dfrac{L_i + L_j}{2} + a|i-j| + 2v & \text{if } L_i + L_j < (d_v - a)|i-j| + 2(w-v), \\ \\ (E[Z_{ij}|Z_{ij} \leq q_{ij}] & \\ +a|i-j| + 2v)Pr(Z_{ij} \leq q_{ij}) & \\ +(L_i + L_j - E[Z_{ij}|Z_{ij} > q_{ij}] & \\ +d_v|i-j| + 2w)\,Pr(Z_{ij} > q_{ij}) & \text{otherwise.} \end{cases} \tag{6}$$

As discussed earlier, the results for Region 2 can be extended to Regions 1 and 3. The number of aisles and the associated aisle lengths are different; however, the equations presented in Section 3.1.1 for q_{ij}, $E[Z|Z \leq q_{ij}]$, $E[Z|Z > q_{ij}]$ and $Pr(Z \leq q_{ij})$ are also valid for Regions 1 and 3. The expected distance between two locations within either Region 1 or 3 can be obtained from (6) simply by replacing the term d_v with d_h.

From probability theory, we know the expected distance between two uniformly distributed locations on the same aisle of length L_i is $L_i/3$. We define $i, j = 1, ..., N_r$ for Region r, where $r = 1$, 2 or 3. Given that a location is in Region r, the probability of it occurring on aisle i is

$$p_i = \frac{L_i}{\sum_{j=1}^{N_r} L_j}, \tag{7}$$

and

$$\sum_i p_i^2 + \sum_i \sum_{j \neq i} p_i p_j = 1.$$

The expected travel distance between two locations in Region r is then

$$E[TB^{rr}] = \sum_i \left(\frac{L_i}{3}\right) p_i^2 + \sum_i \sum_{j \neq i} E[TB^{rr}_{ij}|i \neq j] p_i p_j. \tag{8}$$

3.2 Travel Between Regions 1 and 2

There are two cases to consider when the travel is between a location in Region 1 and a location in Region 2. These two cases depend upon the relative positions of the picking aisles. Let the location in Region 1 be in aisle i, where $i = 1, ..., N_1$, and let the location in Region 2 be in aisle j, where $j = 1, ..., N_2$. In case A, as shown in Figure 4, aisles i and j meet two conditions: they are both on the left side of the warehouse, and the intersection of aisle i with the diagonal cross aisle is closer to the P&D point than the intersection of aisle j with the cross aisle. When these two conditions exist, travel uses either the left vertical cross aisle (with aisle j' or j''), or the diagonal cross aisle (see Figure 4).

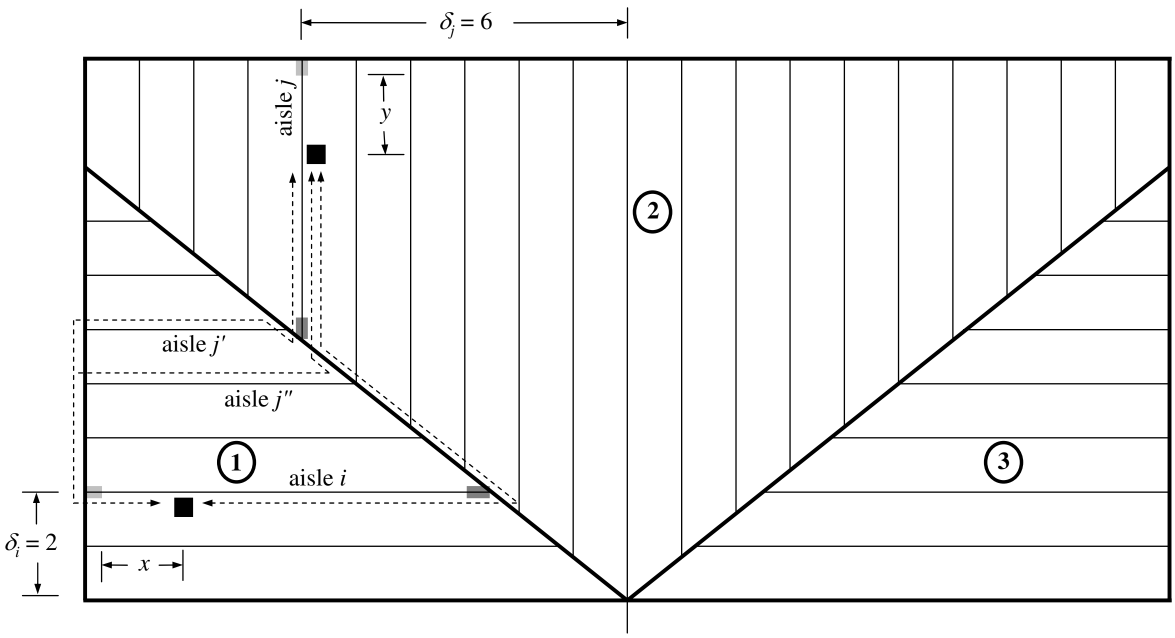


Figure 4: Travel between a location in Region 1 and location in Region 2, case A (δ_i and δ_j are defined and used in Appendix B).

Case B applies to all aisle combinations that do not meet the conditions of case A. This includes aisles i and j on the left side of the warehouse, but the intersection of aisle j with the cross aisle is closer to the P&D point (or the same distance). Case B also includes the instances for which aisles i and j are on opposite sides of the warehouse. For case B, travel uses either the top horizontal cross aisle (with aisle i' or aisle i'') or the diagonal cross aisle (see Figure 5).

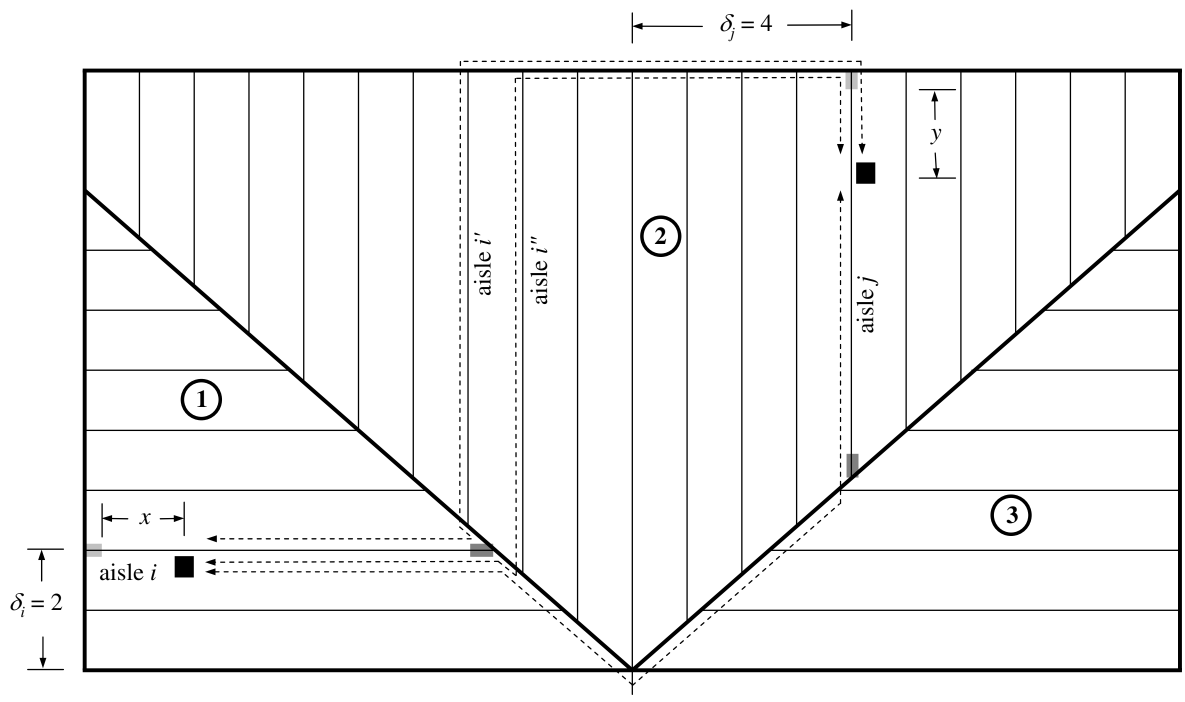


Figure 5: Travel between a location in Region 1 and a location in Region 2, case B (δ_i and δ_j are defined and used in Appendix B).

Let A indicate the set of aisle pairs that meet the conditions of case A, and B indicate the set of aisle pairs that meet the conditions of case B. Since the sets A and B partition the set of all aisle pairs between Regions 1 and 2, the total expected distance between a location in Region 1 and a location in Region 2 is

$$E[TB^{12}] = \sum_{(i,j)\in A} E[TB_{ij}^{12(A)}]p_i p_j + \sum_{(k,l)\in B} E[TB_{kl}^{12(B)}]p_k p_l, \tag{9}$$

where p_i is defined by (7). See (17) and (22) in Appendix B for the expected travel-between for case A, $E[TB_{ij}^{12(A)}]$, and the expected travel-between for case B, $E[TB_{kl}^{12(B)}]$, respectively. Because our warehouse is symmetric, (9) also represents the expected distance between a location in Region 2 and a location in Region 3, i.e.,

$$E[TB^{23}] = E[TB^{12}]. \tag{10}$$

3.3 Travel Between Regions 1 and 3

For travel between Regions 1 and 3, let one location be in aisle i of Region 1 and the other location be in aisle j of Region 3, where $i, j = 1, ..., N_1$ and $N_1 = N_3$. As shown in Figure 6, travel between locations uses either the diagonal cross aisle or the top horizontal cross aisle. If travel is along the top horizontal cross aisle, there are potentially two alternative paths from aisle i to the top cross aisle (aisle i' or i''), and two alternative paths from aisle j to the top cross aisle (aisle j' or j''). The expected travel between locations in Regions 1 and 3 is

$$E[TB^{13}] = \sum_i \sum_j E[TB_{ij}^{13}] p_i p_j, \tag{11}$$

where p_i is defined by (7). See (23) in Appendix C for the derivation of $E[TB_{ij}^{13}]$.

3.4 Total Expected Travel Distance Between Two Locations

The probability of two locations of a dual-command cycle being in Regions r and s ($r, s =$ 1, 2, or 3) is

$$w_{rs} = \frac{T_r T_s}{T^2},$$

where T_r is the total length of picking aisles in Region r, T_s is the total length in Region s, and T is the total length in the warehouse. Using these probabilities and (8)–(11), the expected travel between locations in a warehouse is

$$\begin{aligned} E[TB] &= E[TB^{11}]w_{11} + E[TB^{22}]w_{22} + E[TB^{33}]w_{33} \\ &\quad + 2E[TB^{12}]w_{12} + 2E[TB^{13}]w_{13} + 2E[TB^{23}]w_{23}. \end{aligned}$$

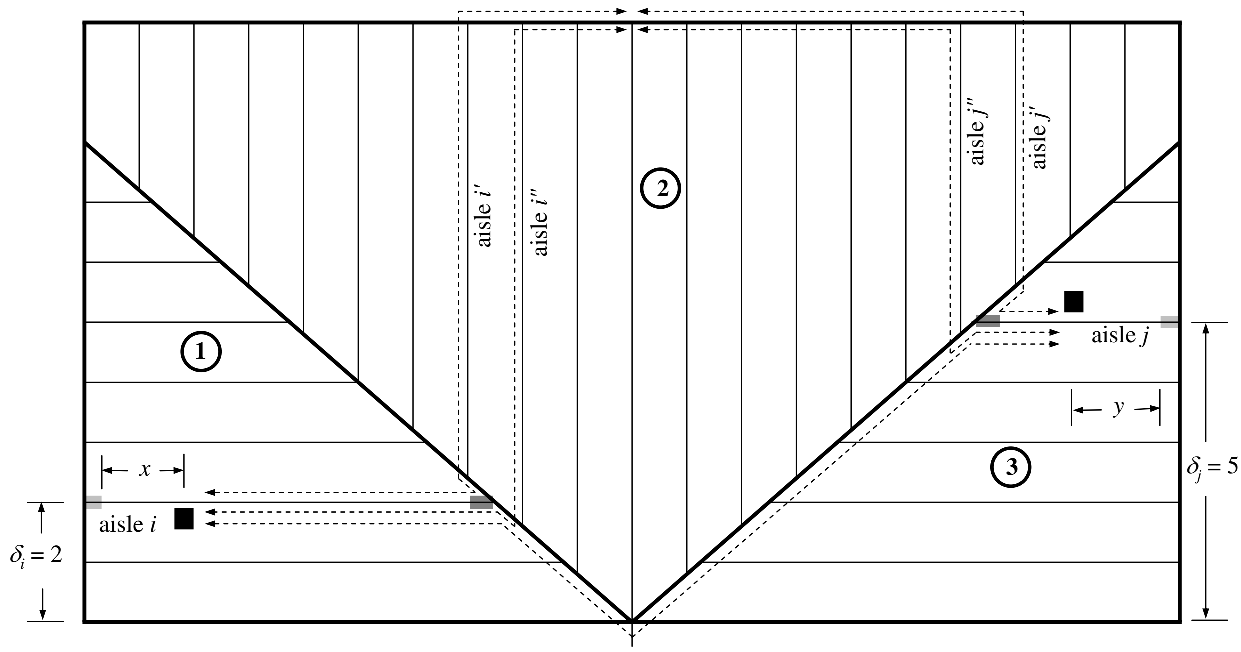


Figure 6: Travel between a location in Region 1 and location in Region 3 (δ_i and δ_j are defined in Appendix C).

4 Optimizing Fishbone Aisles for Dual-Command Operations

To design fishbone warehouses for dual-command operations, we require an analytical expression for dual-command travel distance. Expected dual-command travel distance is the sum of expected single-command travel distance and expected travel-between distance, i.e., $E[DC] = E[SC] + E[TB]$. We use the expressions for expected travel-between distance developed in Section 3, and the one-way travel distance expression developed in Gue and Meller (2008). Gue and Meller (2008) used their one-way travel expression and nonlinear optimization methods to find the optimal slope of the diagonal cross aisle; while we use a discrete grid-based search over a range of cross aisle slopes and warehouse shapes to determine the best fishbone design for a given warehouse size. To facilitate our analysis approach and maintain consistent notation with Section

3, we rewrite the original equations for one-way travel distance, add a distance term of $a/2$ to model the distance traveled into and out of the picking space, and multiply the result by two to yield a single-command cycle distance. The expected single-command travel distance is

$$E[SC] = 2\left[\frac{a}{2} + \sum_{i\in V} p_i\left(\delta_i d_v + w + \frac{L_i}{2}\right) + \sum_{j\in H} p_j\left(\delta_j d_h + w + \frac{L_j}{2}\right)\right],$$

where V is the set of all vertical aisles (Region 2) and H is the set of all horizontal aisles (Regions 1 and 3). δ_i and δ_j represent the integer number of aisle widths between the P&D point and aisles i and j, respectively.

Using these analytical expressions, we can now calculate $E[DC]$ for any fishbone warehouse. We consider a range of warehouse sizes, where the size (or storage capacity) of a warehouse is measured by the total length of the picking aisles, T. The best fishbone aisle design for a given T minimizes $E[DC]$ and has a particular shape and aisle structure. To find this best design, we vary two parameters: warehouse width, as defined by the number of vertical aisles in Region 2, and the slope of the diagonal cross aisle. Because we maintain a constant T, varying the warehouse width results in a variety of warehouse shapes. The grid-search is conducted as follows:

1. For each value of T, enumerate the meaningful warehouse widths over a discrete number of vertical aisles.

2. For each resulting width, consider 100 slope values between the minimum slope of zero and the maximum slope that occurs when the diagonal cross aisle meets the upper corners of the warehouse.

3. Calculate $E[DC] = E[SC] + E[TB]$ for each point in the grid.

4. Select the warehouse width and diagonal aisle slope that results in the

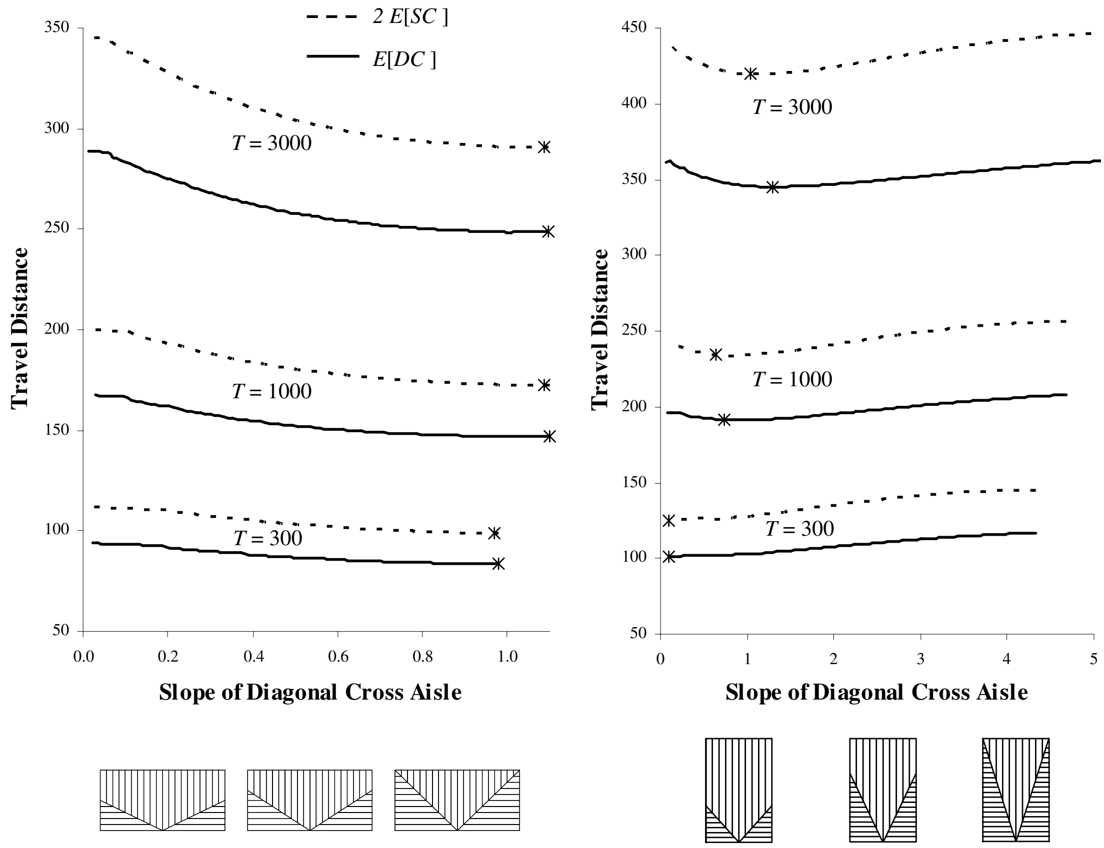
minimum value of $E[DC]$.

We assume square pallet footprints (which include clearances), and specify the warehouse dimensions in pallets, where the center-to-center distance between adjacent picking aisles, a, is 5 pallets, and the cross aisle width, $2v$, is 3 pallets. As mentioned in Section 3, a warehouse with a total picking aisle length $T = 1000$ pallets implies a warehouse with 1000 "locations," but 2000 columns of pallet positions.

Some of the results from the optimization with $T = 300$ are shown in Table 1 in Appendix D. Although 100 slopes were considered for each warehouse width, for brevity, Table 1 shows only 13 of the 100 slopes considered (including the maximum and minimum slopes for each width). Note that for $T = 300$, the minimum $E[DC]$ occurs when the warehouse width corresponds to 13 vertical aisles and the slope is at its maximum of 0.98 (shaded region in table).

Through this analysis, we gained several insights, or rules-of-thumb, for designing dual-command warehouses with fishbone aisles. For example, the best cross aisle slope is dependent on the shape of the warehouse. Figure 7 shows the results for two general warehouse shapes and three different sizes ($T = 300$, $T = 1000$ and $T = 3000$). For warehouses that are approximately twice as wide as they are tall (Figure 7(a)) the best slope (indicated with an asterisk *) is at, or close to, its maximum possible value. However, for tall, narrow warehouses (Figure 7(b)) with relatively few, long vertical aisles, the best slope is typically much lower than its maximum possible value. Notice that the curves for $E[DC]$ and $2E[SC]$ are similar in shape, indicating that the best slope for single-command travel distance is also nearly the best for dual-command travel distance.

Figure 8 shows $E[DC]$ and $E[SC]$ plotted versus warehouse width for three values of T. For each of the warehouse widths evaluated, the best cross aisle slope was chosen. We note that for the three warehouse sizes shown ($T = 300$, 1000 and 3000), $E[DC]$ and $E[SC]$ are both near their minimum when the number of vertical aisles is equal to 13, 21 and 35, respectively (indicated by the vertical



(a) Wide Fishbone Warehouses (b) Tall Fishbone Warehouses

Figure 7: $E[DC]$ and $2E[SC]$ versus slope for $T = 300$, $T = 1000$, $T = 3000$; (a) Fishbone warehouses that are approximately twice as wide as they are tall; (b) Fishbone warehouses that are approximately twice as tall as they are wide.

dashed lines). These widths correspond to warehouses that are approximately twice as wide as they are tall, resulting in half-warehouses that are approximately square. This result is consistent with research on optimal shapes for single-command travel in traditional warehouses (Francis, 1967; Bassan et al., 1980; Pohl et al., 2008).

From the results of this empirical study, we suggest two design rules for fishbone warehouses that perform dual-command operations. The best, or nearly-best, design is obtained by (1) choosing a warehouse shape that is approximately a square half-warehouse (Figure 8), and (2) extending the diagonal cross aisle to the upper corners of the picking space (Figure 7(a)). Another important re-

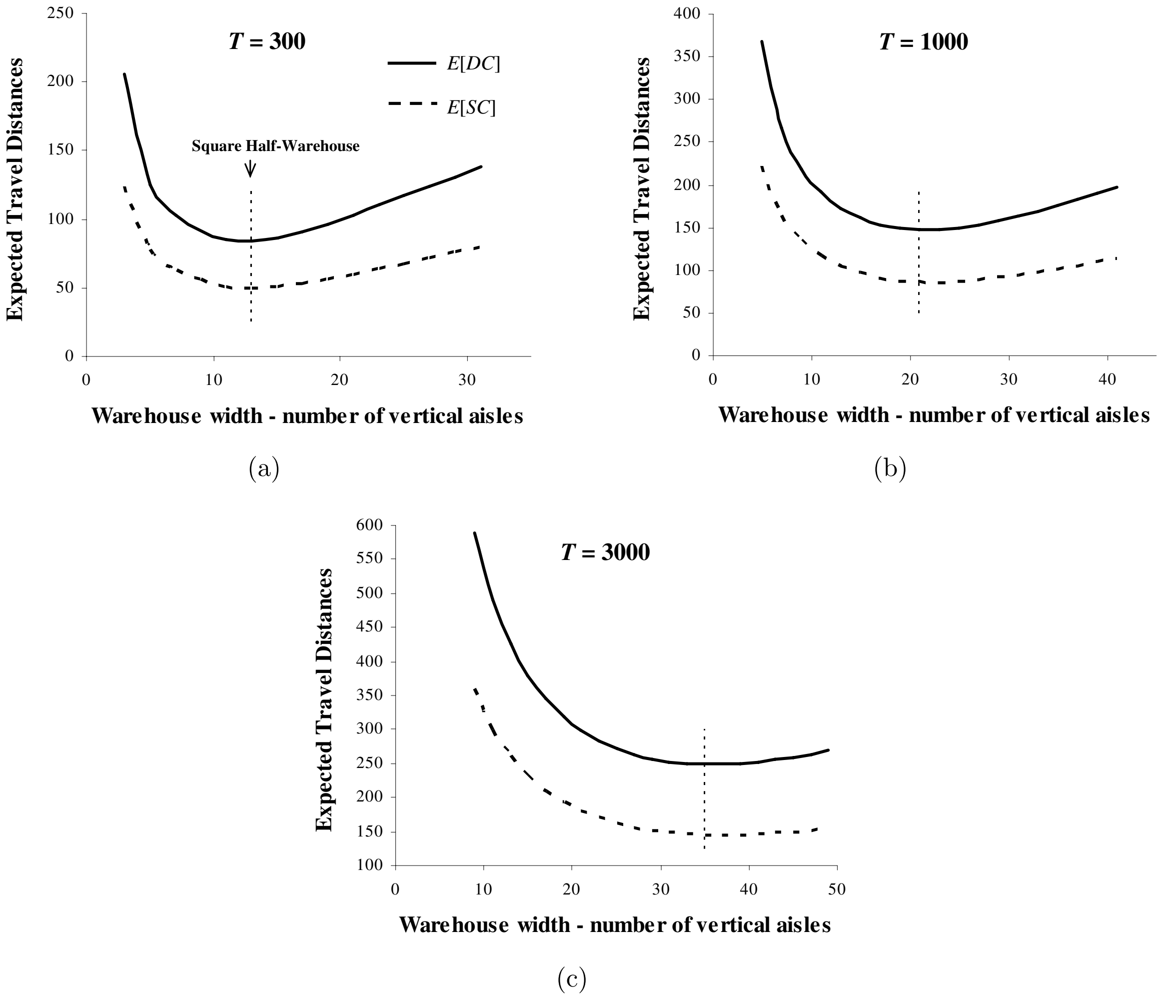


Figure 8: Optimal Warehouse Shape for $E[DC]$ and $E[SC]$ for (a) $T = 300$, (b) $T = 1000$ and (c) $T = 3000$

sult from this study is that a fishbone warehouse that has been optimized for single-command operations is also near-optimal for dual-command operations. The robustness of the design to these two operating modes is beneficial, since many warehouses perform a combination of single- and dual-command cycles.

5 Comparison of Fishbone to Traditional Warehouses

In this section we compare fishbone warehouses to the traditional warehouses that we designated in Figure 1 as Layouts A and B. For simplicity, we do not include Layout C in the comparison since its dual-command performance is very similar to that of Layout B (Pohl, Meller and Gue, 2008). Note that for the same total picking aisle length T, the warehouses will have slightly different areas. For the same width, a fishbone warehouse and Layout B are larger than Layout A, because of the area required by the middle aisle.

Figure 9 compares travel distance in warehouses with $T = 1000$ for the fishbone design and the two traditional aisle designs. The diagonal aisle slope for each fishbone warehouse was selected to minimize expected dual-command travel distance. Figure 9(a) is similar to the results of Gue and Meller (2008), in that the fishbone design improves over both traditional designs for single-command travel (except for relatively tall, narrow warehouses, where the performance is similar). Note that the optimal $E[SC]$ for all three warehouses occurs when the warehouse is approximately twice as wide as it is tall (21 aisles).

Figure 9(b) illustrates that a fishbone warehouse has lower expected travel distance between two locations than the traditional warehouse without a middle cross aisle for all warehouse widths considered. However, a fishbone warehouse has a higher expected travel distance between locations when compared to a warehouse with a standard middle cross aisle, particularly for warehouses that are taller than they are wide. This is a significant result since it indicates that the average improvement for dual-command travel for fishbone warehouses will likely be less than the improvement previously illustrated for single-command travel. And although we do not investigate the performance of fishbone warehouse for the general order picking problem (with more than two locations visited per tour), this result indicates that the performance of fishbone warehouses are likely to be poor in such an environment.

For dual-command travel, we see in Figure 9(c) that the fishbone design minimizes the expected dual-command travel distance in warehouses ($T = 1000$) that are 15 aisles wide or wider, whereas the warehouse with a conventional middle cross aisle has the lowest expected travel for warehouses with fewer than 15 aisles. For $T = 1000$ and $n = 15$, the warehouses are approximately square (i.e., the half-warehouses are approximately twice as tall as they are wide). That is, the point where fishbone dominates the traditional warehouses occurs when the warehouses are approximately square. A similar analysis is shown in Figure 10

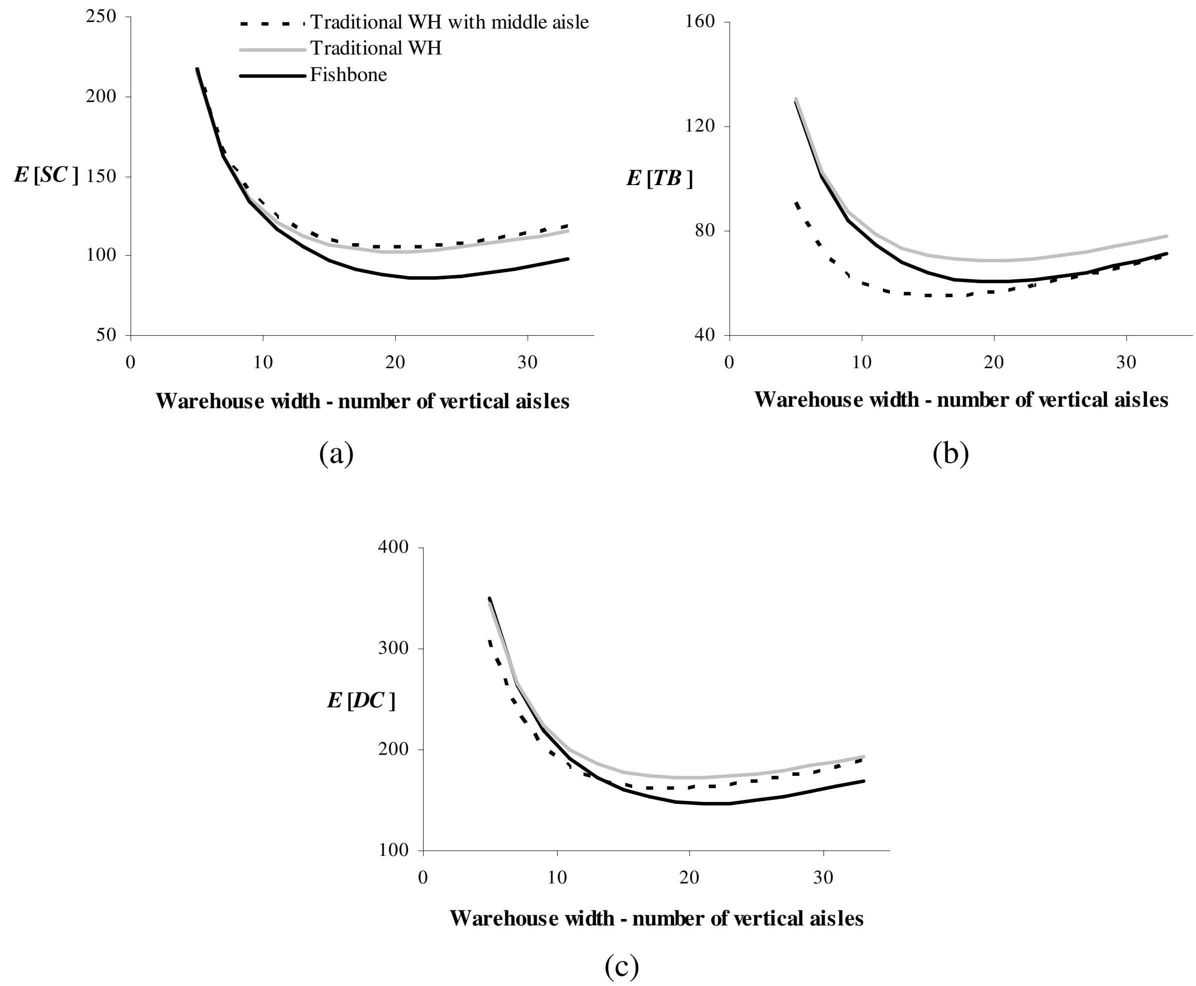


Figure 9: Fishbone design and equivalent traditional designs, $T = 1000$, (a) Single-command travel distance, (b) Travel-between, (c) Dual-command travel distance.

for four other warehouse sizes, where for $T = 300$, 500, 2000 and 3000, the number of aisles that correspond to approximately square warehouses are 9, 11, 21 and 25, respectively (as indicated by the solid vertical lines).

The results in Figure 10 suggest a general design rule: For existing facilities where the shape of the warehouse is fixed (and aisle widths are similar to our assumptions) the choice of aisle design depends on the shape. For warehouses that are taller than they are wide, the conventional perpendicular cross aisle is probably best, because it provides significant improvement for travel-between. However, for warehouses that are wider than they are tall, the fishbone design is preferred because the expected single-command travel distance

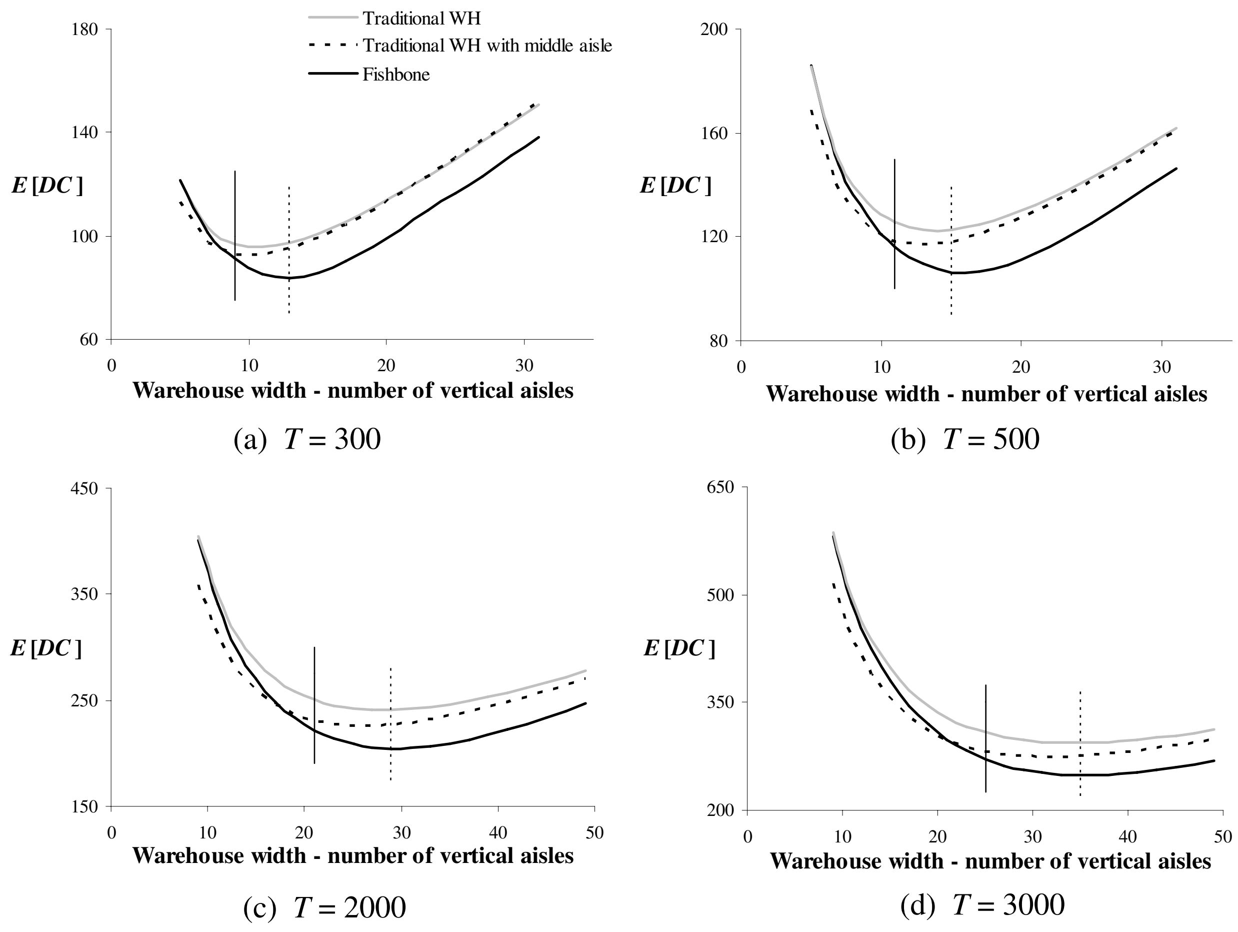


Figure 10: $E[DC]$ for fishbone designs and equivalent traditional designs.

is much lower and the perpendicular cross aisle does not provide as much advantage for travel-between over a fishbone aisle design. As discussed earlier, the optimal fishbone design for dual-command travel is an approximately square half-warehouse, which is illustrated in Figure 10 with dashed vertical lines.

When designers have control over the shape of the storage area, as in a greenfield design, the fishbone design is always better than both of the traditional designs. Figure 11(a) shows the improvement of optimal fishbone warehouses over optimal traditional designs for a range of warehouse sizes. For all values of T, the fishbone design improves upon the traditional warehouse with a middle cross aisle by approximately 10%, and improves upon the traditional warehouse by up to 15.5%. The fishbone warehouses evaluated in Figure 11(a) are larger than the conventional warehouses, as indicated in Figure 11(b). Notice that the curves are not smooth, due to the discrete changes in the number of aisles in each design. The comparison with the traditional warehouse in Figure 11(a) illustrates the decreasing influence of the space consumed by the cross aisle as the warehouse increases in size (i.e., the percent improvement of the optimized fishbone warehouse improves as the warehouse size increases, because the negative effect of the additional cross aisle on space utilization decreases as the warehouse size increases). **The values used to generate Figures 11(a) and 11(b) can be found in Table 2, Appendix D.**

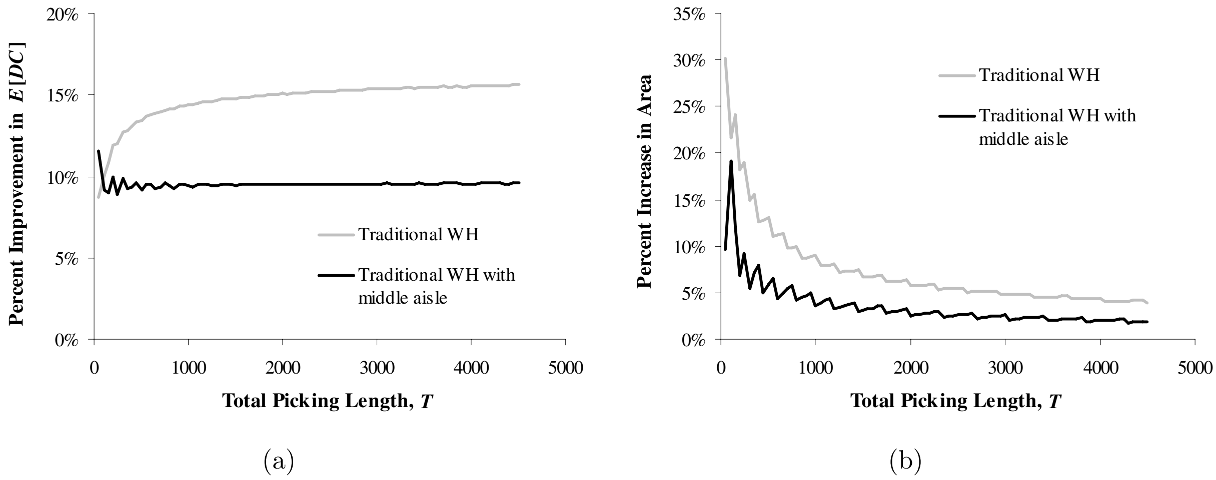


Figure 11: (a) Percent improvement of optimal fishbone designs over optimal traditional designs for dual-command travel distance; (b) Percent increase in area for optimal fishbone designs over optimal traditional designs.

6 Conclusions & Future Research

Gue and Meller (2006) proposed the fishbone aisle design and showed that up to a 20% reduction in single-command travel distance is possible, as compared to a traditional warehouse with parallel aisles. We extend their work by developing an analytical expression for the expected travel distance between locations in a fishbone warehouse. Combining this expression with the existing expression for single-command travel distance allows us to optimize fishbone aisle designs for dual-command use, and to make a thorough comparison with traditional warehouses.

We compare dual-command travel distance in fishbone warehouses to dual-command travel distance in traditional warehouses with the same total picking aisle length. We optimize the shape of each warehouse by choosing the number of aisles (number of vertical aisles in the fishbone warehouses) that minimizes dual-command travel distance. For the fishbone designs, we also orient the diagonal aisles in a way that minimizes dual-command travel distance by choosing the best slope from a set of discrete possibilities.

Our main result is that

> *A properly-designed fishbone warehouse reduces dual-command travel by approximately 10%–15%.*

That is, when designers have control over the shape of the warehouse, a fishbone design can always be found that improves upon the traditional designs. A fishbone warehouse will reduce dual-command travel by almost 10%, when compared to a traditional warehouse with a middle cross aisle, and by up to 15.5% when compared to a traditional warehouse without a middle aisle. We note that the latter improvement comes at a "cost" of a facility that is approximately 5% larger.

A secondary result relates to two design rules for fishbone warehouses that perform dual-command operations.

> ***The best, or nearly-best, design is obtained by (1) choosing a warehouse shape that is approximately a square half-warehouse, and (2) extending the diagonal cross aisle to the upper corners of the picking space.***

Such a design is also best or nearly-best when a warehouse is performing single-command operations.

We also compare warehouses with the same total picking aisle length over a variety of warehouse shapes. When the shape of the warehouse area is given, such as in an existing facility, we see that for warehouses that are taller than they are wide, a traditional warehouse with a middle cross aisle is preferred. However, for warehouses that are wider than they are tall, the fishbone design provides the shortest travel distances.

Our research can be extended in several directions. Because some unit-load warehouses do not use randomized storage, the performance of fishbone designs under other storage policies can be investigated. Possibilities include storage policies based on relative product velocities, such as full-turnover and class-based turnover strategies.

We are also considering new aisle layouts that are designed particularly for dual-command travel. There may exist entirely new designs that perform better than the fishbone design for dual-command operations. Finally, new designs should be developed for the general order picking problem, with more than two locations visited per tour. We believe this is a very challenging problem, whose solution would find many applications in industry.

Acknowledgements

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Appendices (intended as an online supplement)

A Travel Within a Region

A.1 Distribution of Z

We assume the storage and retrieval activities are distributed uniformly along each aisle length, therefore $X_i \sim U(0, L_i)$ and $Y_j \sim U(0, L_j)$. We let $Z_{ij} = X_i + Y_j$, and use convolution to determine the probability density function of Z_{ij}. If aisles i and j are the same length, the density of Z_{ij} is triangular. If the aisle lengths are different, with $L_j > L_i$, the density of Z_{ij} is an isosceles trapezoid (Killmann and von Collani, 2001), as shown in Figure 12, and expressed as

$$f_Z(z) = \begin{cases} \dfrac{z}{L_i L_j} & \text{for } 0 \leq z < L_i, \\[2ex] \dfrac{1}{L_j} & \text{for } L_i \leq z < L_j, \\[2ex] \dfrac{L_i + L_j - z}{L_i L_j} & \text{for } L_j \leq z \leq L_i + L_j. \end{cases}$$

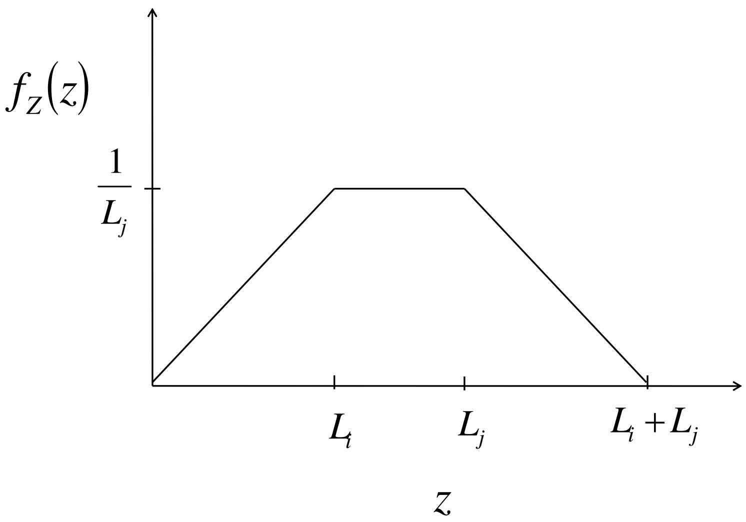

Figure 12: Probability density function of $Z_{ij} = X_i + Y_j$ when $X_i \sim U(0, L_i)$ and $Y_j \sim U(0, L_j)$.

The cumulative distribution for Z_{ij} is found by using the relationship $F_Z(z) = \int_0^z f_Z(u)\,du$. For the interval $0 \le z < L_i$,

$$F_Z(z) = \int_0^z \frac{u}{L_i L_j}\,du = \frac{z^2}{2L_i L_j}.$$

If $L_i \le z < L_j$,

$$F_Z(z) = F_Z(L_i^-) + \int_{L_i}^z \frac{1}{L_j}\,du = \frac{L_i}{2L_j} + \frac{z - L_i}{L_j} = \frac{2z - L_i}{2L_j}.$$

If $L_j \le z < L_i + L_j$,

$$\begin{aligned}
F_Z(z) &= F_Z(L_j^-) + \int_{L_j}^z \frac{L_i + L_j - u}{L_i L_j}\,du \\
&= \frac{2L_j - L_i}{2L_j} + \frac{1}{L_i L_j}\left[(L_i + L_j)(z - L_j) - \frac{1}{2}(z^2 - L_j^2)\right] \\
&= \frac{-z^2 + 2(L_i + L_j)z - L_i^2 - L_j^2}{2L_i L_j} = \frac{2L_i L_j - (z - L_i - L_j)^2}{2L_i L_j}.
\end{aligned}$$

The resulting cumulative distribution function is

$$F_Z(z) = \begin{cases} 0 & \text{for } z < 0, \\ \dfrac{z^2}{2L_i L_j} & \text{for } 0 \le z < L_i, \\ \dfrac{2z - L_i}{2L_j} & \text{for } L_i \le z < L_j, \\ \dfrac{2L_i L_j - (z - L_i - L_j)^2}{2L_i L_j} & \text{for } L_j \le z < L_i + L_j, \\ 1 & \text{for } z \ge L_i + L_j. \end{cases}$$

A.2 Distribution of Z Conditioned on q_{ij}

We find $f_{Z|Z\leq q_{ij}}$ and $f_{Z|Z>q_{ij}}$ by using the relationships

$$F_{Z|Z\leq q_{ij}} = Pr(Z_{ij} \leq z | Z_{ij} \leq q_{ij}) = \frac{Pr(Z_{ij} \leq z, Z_{ij} \leq q_{ij})}{Pr(Z_{ij} \leq q_{ij})} = \frac{F_Z(z)}{F_Z(q_{ij})} \tag{12}$$

and

$$\begin{aligned} F_{Z|Z>q_{ij}} &= Pr(Z_{ij} \leq z | Z_{ij} > q_{ij}) = 1 - Pr(Z_{ij} > z | Z_{ij} > q_{ij}) \\ &= 1 - \frac{Pr(Z_{ij} > z, Z_{ij} > q_{ij})}{Pr(Z_{ij} > q_{ij})} = 1 - \frac{1 - F_Z(z)}{1 - F_Z(q_{ij})} \\ &= \frac{F_Z(z) - F_Z(q_{ij})}{1 - F_Z(q_{ij})}, \end{aligned} \tag{13}$$

and differentiating. Since F_Z is a piecewise function, we must derive $f_{Z|Z\leq q_{ij}}$ and $f_{Z|Z>q_{ij}}$ for three intervals of q_{ij}. This process, while not difficult, is tedious, therefore we provide the derivations for only the first interval, $0 < q_{ij} < L_i$, and simply present the results for the two remaining intervals, $L_i \leq q_{ij} < L_j$ and $L_j \leq q_{ij} \leq L_i + L_j$.

If $0 < q_{ij} < L_i$, then to calculate $f_{Z|Z\leq q_{ij}}$ we need only consider the first interval of F_Z, since Z_{ij} can be less than q_{ij} only in that interval. Therefore, using (12),

$$F_{Z|Z\leq q_{ij}} = \frac{F_Z(z)}{F_Z(q_{ij})} = \frac{z^2}{2L_iL_j}\left(\frac{2L_iL_j}{q_{ij}^2}\right) = \frac{z^2}{q_{ij}^2},$$

and

$$f_{Z|Z\leq q_{ij}}(z) = \frac{d}{dz} f_{Z|Z\leq q_{ij}}(z) = \frac{2z}{q_{ij}^2}.$$

To derive $f_{Z|Z>q_{ij}}$ when $0 < q_{ij} < L_i$, we must consider all three intervals of F_Z, since it is possible that $Z_{ij} > q_{ij}$ in any of the three intervals. $F_Z(z)$ is different for each interval, however $F_Z(q_{ij})$ does not change, and $1 - F_Z(q_{ij}) = (2L_iL_j - q_{ij}^2)/(2L_iL_j)$. Using (13) we find that for $q_{ij} < z < L_i$,

$$F_{Z|Z \geq q_{ij}} = \frac{F_Z(z) - F_Z(q_{ij})}{1 - F_Z(q_{ij})} = \left(\frac{z^2}{2L_iL_j} - \frac{q_{ij}^2}{2L_iL_j} \right) \left(\frac{2L_iL_j}{2L_iL_j - q_{ij}^2} \right) = \frac{z^2 - q_{ij}^2}{2L_iL_j - q_{ij}^2}. \quad (14)$$

For $L_i \leq z < L_j$,

$$F_{Z|Z \geq q_{ij}} = \left(\frac{2z - L_i}{2L_j} - \frac{q_{ij}^2}{2L_iL_j} \right) \left(\frac{2L_iL_j}{2L_iL_j - q_{ij}^2} \right) = \frac{2L_iz - L_i^2 - q_{ij}^2}{2L_iL_j - q_{ij}^2}, \quad (15)$$

and for $L_j \leq z < L_i + L_j$,

$$F_{Z|Z \geq q_{ij}} = \left(\frac{2L_iL_j - (z - L_i - L_j)^2}{2L_iL_j} - \frac{q_{ij}^2}{2L_iL_j} \right) \left(\frac{2L_iL_j}{2L_iL_j - q_{ij}^2} \right) = 1 - \frac{(z - L_i - L_j)^2}{2L_iL_j - q_{ij}^2}. \quad (16)$$

When we differentiate (14)–(16) with respect to z, the resulting density function is

$$f_{Z|Z > q_{ij}}(z) = \begin{cases} \dfrac{z}{L_iL_j - q_{ij}^2/2} & \text{for } q_{ij} < z < L_i, \\ \\ \dfrac{L_i}{L_iL_j - q_{ij}^2/2} & \text{for } L_i \leq z < L_j, \\ \\ \dfrac{L_i + L_j - z}{L_iL_j - q_{ij}^2/2} & \text{for } L_j \leq z \leq L_i + L_j. \\ \end{cases}$$

For the remaining two intervals of q_{ij}, we present the resulting density functions, but leave the derivation to the reader. If $L_i \leq q_{ij} < L_j$, then

$$f_{Z|Z \leq q_{ij}}(z) = \begin{cases} \dfrac{z}{L_i(q_{ij} - L_i/2)} & \text{for } 0 \leq z < L_i, \\ \\ \dfrac{1}{q_{ij} - L_i/2} & \text{for } L_i \leq z \leq q_{ij}, \end{cases}$$

and

$$f_{Z|Z>q}(z) = \begin{cases} \dfrac{1}{L_i/2 + L_j - q_{ij}} & \text{for } q_{ij} \le z < L_j, \\[2em] \dfrac{L_i + L_j - z}{L_i(L_i/2 + L_j - q_{ij})} & \text{for } L_j \le z \le L_i + L_j. \end{cases}$$

If $L_j \le q_{ij} \le L_i + L_j$, then

$$f_{Z|Z\le q_{ij}}(z) = \begin{cases} \dfrac{z}{L_iL_j - (q_{ij} - L_i - L_j)^2/2} & \text{for } 0 \le z < L_i, \\[2em] \dfrac{L_i}{L_iL_j - (q_{ij} - L_i - L_j)^2/2} & \text{for } L_i \le z \le L_j, \\[2em] \dfrac{L_i + L_j - z}{L_iL_j - (q_{ij} - L_i - L_j)^2/2} & \text{for } L_j \le z \le q_{ij}, \end{cases}$$

and

$$f_{Z|Z>q_{ij}}(z) = \frac{L_i + L_j - z}{(q_{ij} - L_i - L_j)^2/2}.$$

B Travel Between Regions 1 and 2

There are two cases to consider when the travel is between a location in Region 1 and a location in Region 2. These two cases depend upon the relative positions of the picking aisles and will be discussed in the following sections. As shown in Figure 4, we let δ_i represent the integer number of aisle widths between the bottom picking aisle and aisle i, and let δ_j represent the integer number of aisle widths between the center vertical aisle in Region 2 and aisle j. The distance between aisles i and j along the diagonal cross aisle is $|\delta_j d_v - \delta_i d_h|$.

B.1 Case A

In case A (Figure 4) aisles i and j meet two conditions: they are both on the left side of the warehouse, and the intersection of aisle i with the diagonal cross aisle is closer to the P&D point than the intersection of aisle j with the cross aisle. When these two conditions are met, the shortest path uses either the left vertical cross aisle or the diagonal cross aisle. When the left vertical cross aisle is used to reach aisle j, the worker traverses the vertical cross aisle and then one of two picking aisles in Region 1, j' or j''. Aisle j' in Figure 4 intersects the diagonal cross aisle at the same point as aisle j, or slightly farther up the cross aisle, while aisle j'' intersects the diagonal cross aisle slightly below aisle j. To determine the difference between these two options, we define P_{ij} as the minimum of the paths that use aisles j' and j'',

$$P_{ij} = \min \{ (j' - i)a + L_{j'} + \delta_{j'} d_h - \delta_j d_v, (j'' - i)a + L_{j''} + \delta_j d_v - \delta_{j''} d_h \},$$

or equivalently,

$$P_{ij} = \begin{cases} (j' - i)a + L_{j'} + \delta_{j'} d_h - \delta_j d_v & \text{if } 2\delta_j d_v - 2\delta_{j'} d_h > a(1 - 1/m) - d_h, \\ (j'' - i)a + L_{j''} + \delta_j d_v - \delta_{j''} d_h & \text{otherwise.} \end{cases}$$

The shortest distance using the vertical cross aisle is then $x + 2v + P_{ij} + 2w + (L_j - y)$.

The other choice is the path that uses only the diagonal cross aisle, with length $(L_i - x) + 2w + \delta_j d_v - \delta_i d_h + (L_j - y)$. We are indifferent about which path to choose when they are the same length. That is, when

$$x + 2v + 2w + P_{ij} + (L_j - y) = (L_i - x) + 2w + \delta_j d_v - \delta_j d_h + (L_j - y),$$

and

$$x = \frac{1}{2} [L_i - P_{ij} + \delta_j d_v - \delta_i d_h] - v.$$

We see that our choice depends only on the value of x, and is independent of y, since in both paths we enter aisle j from the same end. Let q_{ij}^x be a parameter associated with aisles i and j, where its value is the same as the variable x *only* when the two paths are equivalent in length. That is,

$$q_{ij}^x = \frac{1}{2} [L_i - P_{ij} + \delta_j d_v - \delta_i d_h] - v.$$

If $x = q_{ij}^x$, the alternative paths are the same length (by definition), and we are indifferent about which to choose. If $x < q_{ij}^x$, the shortest path between locations uses the left vertical cross aisle, and if $x > q_{ij}^x$, the shortest path uses the diagonal cross aisle. Since X_i is uniformly distributed, $Pr(x \le q_{ij}^x) = q_{ij}^x / L_i$. For case A, the expected travel between a location in aisle i of Region 1 and a location in aisle j of Region 2 is

$$\begin{aligned} E[TB_{ij}^{12(A)}] &= \left[E[X|X \le q_{ij}^x] + 2(v+w) + P_{ij} + L_j - E[Y]\right] Pr(x \le q_{ij}^x) \\ &\quad + \left[L_i - E[X|X > q_{ij}^x] + 2w + \delta_j d_v - \delta_i d_h + L_j - E[Y]\right] Pr(x > q_{ij}^x) \\ &= \left[\frac{q_{ij}^x}{2} + 2(v+w) + P_{ij} + L_j - \frac{L_j}{2}\right] \left(\frac{q_{ij}^x}{L_i}\right) \\ &\quad + \left[\frac{L_i - q_{ij}^x}{2} + 2w + \delta_j d_v - \delta_i d_h + L_j - \frac{L_j}{2}\right] \left(1 - \frac{q_{ij}^x}{L_i}\right) \\ &= \left[\frac{1}{2}(L_j + q_{ij}^x) + 2(v+w) + P_{ij}\right] \left(\frac{q_{ij}^x}{L_i}\right) \\ &\quad + \left[\frac{1}{2}(L_i + L_j - q_{ij}^x) + 2w + \delta_j d_v - \delta_i d_h\right] \left(1 - \frac{q_{ij}^x}{L_i}\right) . \end{aligned} \tag{17}$$

B.2 Case B

Case B applies to all aisle combinations that do not meet the conditions of case A. This includes aisles i and j that share the same side of the warehouse, but the intersection of aisle j with the cross aisle is closer to the P&D point (or the same distance). Case B also includes the instances when aisles i and j are on opposite sides of the warehouse. The potential shortest paths between locations for case B are illustrated in Figure 5. When the top horizontal cross aisle is used to reach aisle j, the worker traverses one of two picking aisles in Region 2, i' or i'', and then the horizontal cross aisle. Aisle i' in Figure 5 intersects the diagonal cross aisle at the same point as aisle i, or slightly farther up the cross aisle, while aisle i'' intersects the diagonal cross aisle slightly below aisle i. To determine the difference between these two options, we define P_{ij} as

$$P_{ij} = \min\left\{\delta_{i'} d_v - \delta_i d_h + L_{i'} + a(j - i'), \delta_i d_h - \delta_{i''} d_v + L_{i''} + a(j - i'')\right\},$$

or equivalently,

$$P_{ij} = \begin{cases} \delta_{i'} d_v - \delta_i d_h + L_{i'} + a(j - i') & \text{if } 2\delta_i d_h - 2\delta_{i'} d_v > a(1-m) - d_v, \\ \delta_i d_h - \delta_{i''} d_v + L_{i''} + a(j - i'') & \text{otherwise.} \end{cases}$$

The shortest distance using the perpendicular cross aisle is then $(L_i - x) + 2w + P_{ij} + 2v + y$. The other choice is the path which uses only the diagonal cross aisle. If aisles i and j are on opposite sides, as shown in Figure 5, the length is $(L_i - x) + 2w + \delta_i d_h + \delta_j d_v + (L_j - y)$. If aisles i and j are on the same side, the length is $(L_i - x) + 2w + \delta_i d_h - \delta_j d_v + (L_j - y)$. We define D_{ij} as the distance along the diagonal cross aisle,

$$D_{ij} = \begin{cases} \delta_i d_h + \delta_j d_v & \text{if } i, j \text{ are on opposite sides,} \\ \delta_i d_h - \delta_j d_v & \text{if } i, j \text{ are on the same side.} \end{cases}$$

The total distance between locations using the diagonal cross aisle is then $(L_i - x) + 2w + D_{ij} + (L_j - y)$. All potential paths access aisle i from the same end, therefore the choice of paths is independent of x.

There are aisle combinations where the shortest travel path always uses one of the two cross aisles, and is independent of both x and y. For example, if aisle i is the topmost aisle in Region 1 and aisle j is the far right aisle in Region 2, for some parameter values, travel-between will always use the top perpendicular cross aisle. In this case,

$$(L_i - x) + 2w + P_{ij} + 2v + y < (L_i - x) + 2w + D_{ij} + (L_j - y) \quad \forall\ 0 \le x \le L_i,\ 0 \le y \le L_j. \tag{18}$$

If (18) holds for $y = L_j$ then it holds for all y, and

$$P_{ij} + 2v + L_j < D_{ij}. \tag{19}$$

When (19) holds, the perpendicular cross aisle will always provide the shortest path, and the expected value of the distance between the two locations is $(L_i + L_j)/2 + P_{ij} + 2v + 2w$.

Conversely, for other aisle combinations, the diagonal cross aisle will always provide the shortest travel distance. For example, if aisle i is the bottom picking aisle in Region 1, then travel to any of the aisles on the right side of Region 2 will use the diagonal cross aisle. For this instance,

$$(L_i - x) + 2w + P_{ij} + 2v + y > (L_i - x) + 2w + D_{ij} + (L_j - y) \qquad \forall\ 0 \le x \le L_i,\ 0 \le y \le L_j. \tag{20}$$

If (20) holds for $y = 0$ then it holds for all y, and

$$P_{ij} + 2v > D_{ij} + L_j. \tag{21}$$

When (21) holds, the diagonal cross aisle will always provide the shortest path, and the expected value of the distance between the two locations is $(L_i + L_j)/2 + D_{ij} + 2w$.

If (19) and (21) do not hold, then the choice of cross aisles depends on y. We are indifferent about which cross aisle to use when the paths are the same length. That is, when

$$(L_i - x) + 2w + P_{ij} + 2v + y = (L_i - x) + 2w + D_{ij} + (L_j - y),$$

and

$$y = \frac{1}{2}\left[L_j + D_{ij} - P_{ij}\right] - v.$$

We define q_{ij}^y, as a parameter of aisles i and j, that equals y *only* when the two paths are equivalent. That is,

$$q_{ij}^y = \frac{1}{2}\left[L_j + D_{ij} - P_{ij}\right] - v.$$

If $y = q_{ij}^y$, the alternative paths are equivalent (by definition). If $y < q_{ij}^y$, the shortest path between locations uses the horizontal cross aisle, and if $y > q_{ij}^y$, the shortest path uses the diagonal cross aisle. Since Y_j is uniformly distributed, $Pr(y \le q_{ij}^y) = q_{ij}^y / L_j$. For case B, the expected travel between a location in aisle i of Region 1 and a location in aisle j of Region 2, when the travel depends on q_{ij}^y, is

$$
\begin{aligned}
E[TB_{ij}^{12(B)}|q_{ij}^y] &= \left[L_i - E[X] + 2w + P_{ij} + 2v + E[Y|Y \le q_{ij}^y]\right] Pr(Y \le q_{ij}^y) \\
&\quad + \left[L_i - E[X] + 2w + D_{ij} + L_j - E[Y|Y > q_{ij}^y]\right] Pr(Y > q_{ij}^y) \\
&= \left[L_i - \frac{L_i}{2} + 2w + P_{ij} + 2v + \frac{q_{ij}^y}{2}\right]\left(\frac{q_{ij}^y}{L_j}\right) \\
&\quad + \left[L_i - \frac{L_i}{2} + 2w + D_{ij} + L_j - \frac{L_j + q_{ij}^y}{2}\right]\left(1 - \frac{q_{ij}^y}{L_j}\right) \\
&= \left[\frac{1}{2}(L_i + q_{ij}^y) + P_{ij} + 2(v+w)\right]\left(\frac{q_{ij}^y}{L_j}\right) \\
&\quad + \left[\frac{1}{2}(L_i + L_j - q_{ij}^y) + D_{ij} + 2w\right]\left(1 - \frac{q_{ij}^y}{L_j}\right) .
\end{aligned}
$$

The expected travel between locations in Regions 1 and 2, for case B, is then

$$
E[TB_{ij}^{12(B)}] = \begin{cases}
\frac{1}{2}(L_i + L_j) + P_{ij} + 2(v+w) & \text{if } P_{ij} + 2v + L_j < D_{ij}, \\
\frac{1}{2}(L_i + L_j) + D_{ij} + 2w & \text{if } P_{ij} + 2v > D_{ij} + L_j, \\
\left[\frac{1}{2}(L_i + q_{ij}^y) + P_{ij} + 2(v+w)\right]\left(\frac{q_{ij}^y}{L_j}\right) & \\
+ \left[\frac{1}{2}(L_i + L_j - q_{ij}^y) + D_{ij} + 2w\right]\left(1 - \frac{q_{ij}^y}{L_j}\right) & \text{otherwise.}
\end{cases} \tag{22}
$$

C Travel Between Regions 1 and 3

As shown in Figure 6, we let δ_i and δ_j be the number of aisle widths between the bottom picking aisle and aisles i and j, respectively. The distance between aisles i and j along the diagonal cross aisle is then $d_h(\delta_i + \delta_j)$. Travel between locations will use either the diagonal cross aisle, or the top horizontal cross aisle. If travel is along the top horizontal cross aisle, there are potentially two alternative paths from aisle i to the top cross aisle, and two alternative paths from aisle j to the top cross aisle. We therefore consider the upper path in two segments, as shown in Figure 6, where the left segment is the path between the location in aisle i and the center of the top cross aisle, and the right segment is the path between the location in aisle j and the center of the top cross aisle. The left segment uses either aisle i' or aisle i'' and is independent of aisle j, so we define P_i as

$$P_i = \min\left\{\delta_{i'} d_v - \delta_i d_h + L_{i'} + ai', \delta_i d_h - \delta_{i''} d_v + L_{i''} + ai''\right\},$$

or equivalently,

$$P_i = \begin{cases} \delta_{i'} d_v - \delta_i d_h + L_{i'} + ai' & \text{if } 2\delta_i d_h - 2\delta_{i'} d_v > a(1-m) - d_v, \\ \delta_i d_h - \delta_{i''} d_v + L_{i''} + ai'' & \text{otherwise.} \end{cases}$$

Similarly, the right segment uses either aisle j' or aisle j'' and is independent of aisle i, so we define P_j as

$$P_j = \begin{cases} \delta_{j'} d_v - \delta_j d_h + L_{j'} + aj' & \text{if } 2\delta_j d_h - 2\delta_{j'} d_v > a(1-m) - d_v, \\ \delta_j d_h - \delta_{j''} d_v + L_{j''} + aj'' & \text{otherwise.} \end{cases}$$

Since we always enter the picking aisles through the same ends, the best cross aisle to use can be determined without knowing the exact locations within their aisles (x and y); we need only know the aisles which contain the two locations. If $P_i + P_j + 2(v+w) < d_h(\delta_i + \delta_j)$ then travel between locations will use the top cross aisle. Otherwise, travel with be along the diagonal cross aisle. The total expected distance between a location in aisle i of Region 1 and a location in aisle j of Region 3 is then

$$E(TB_{ij}^{13}) = \begin{cases} \frac{1}{2}(L_i + L_j) + P_i + P_j + 2v + 4w & \text{if } P_i + P_j + 2(v+w) < d_h(\delta_i + \delta_j), \\ \\ \frac{1}{2}(L_i + L_j) + d_h(\delta_i + \delta_j) + 2w & \text{otherwise}. \end{cases} \tag{23}$$

D Supporting Data Tables

Table 1: Expected Dual-Command Travel Distance for $T = 300$ over a Range of Warehouse Widths and Cross Aisle Slopes

No. Vertical aisles	Cross Aisle Slope	$E[DC]$	No. Vertical aisles	Cross Aisle Slope	$E[DC]$	No. Vertical aisles	Cross Aisle Slope	$E[DC]$	No. Vertical aisles	Cross Aisle Slope	$E[DC]$	No. Vertical aisles	Cross Aisle Slope	$E[DC]$
3	56.17	371.50	9	2.31	93.93	15	0.71	85.96	21	0.34	102.94	27	0.20	126.82
3	51.69	372.93	9	2.12	93.65	15	0.66	85.97	21	0.31	102.44	27	0.18	125.31
3	47.21	371.20	9	1.94	93.27	15	0.60	86.24	21	0.29	102.93	27	0.17	123.52
3	42.73	366.21	9	1.76	92.92	15	0.55	86.97	21	0.26	103.50	27	0.15	124.12
3	38.25	358.09	9	1.57	92.50	15	0.49	87.59	21	0.24	104.18	27	0.14	125.51
3	33.77	346.95	9	1.39	92.01	15	0.44	88.63	21	0.21	104.90	27	0.12	126.81
3	29.29	332.88	9	1.20	91.80	15	0.38	89.59	21	0.19	106.30	27	0.11	127.58
3	24.81	315.98	9	1.02	91.29	15	0.33	90.52	21	0.16	107.59	27	0.09	127.26
3	20.33	296.36	9	0.84	91.37	15	0.27	92.06	21	0.14	108.54	27	0.08	126.85
3	15.85	274.19	9	0.65	91.63	15	0.22	93.33	21	0.11	108.27	27	0.06	128.24
3	11.37	249.49	9	0.47	92.13	15	0.16	94.48	21	0.08	110.17	27	0.05	129.65
3	6.89	221.45	9	0.28	94.10	15	0.11	96.03	21	0.06	111.24	27	0.03	131.09
3	2.41	189.38	9	0.10	93.80	15	0.05	96.76	21	0.03	112.37	27	0.02	132.55
5	10.82	178.32	11	1.44	85.40	17	0.54	90.30	23	0.28	110.13	29	0.17	134.77
5	9.95	176.69	11	1.32	85.24	17	0.50	90.40	23	0.26	110.10	29	0.16	132.06
5	9.09	173.88	11	1.21	85.35	17	0.46	90.75	23	0.24	110.09	29	0.14	131.27
5	8.66	172.34	11	1.16	85.44	17	0.43	90.97	23	0.22	110.17	29	0.14	131.72
5	7.79	168.76	11	1.04	85.41	17	0.39	91.82	23	0.20	110.21	29	0.12	132.63
5	6.93	164.65	11	0.93	85.81	17	0.35	92.85	23	0.18	111.36	29	0.11	133.73
5	6.07	160.05	11	0.82	86.03	17	0.31	93.81	23	0.16	112.84	29	0.10	134.41
5	5.20	155.00	11	0.71	86.53	17	0.27	95.07	23	0.14	114.09	29	0.09	133.99
5	4.34	149.49	11	0.60	87.11	17	0.22	96.32	23	0.12	114.67	29	0.07	132.65
5	3.47	143.54	11	0.48	88.03	17	0.18	97.60	23	0.09	114.95	29	0.06	134.13
5	2.61	137.19	11	0.37	89.13	17	0.14	98.34	23	0.07	116.15	29	0.05	135.64
5	1.75	130.64	11	0.26	90.66	17	0.10	100.07	23	0.05	117.40	29	0.04	137.16
5	0.88	124.88	11	0.15	91.90	17	0.06	100.93	23	0.03	118.69	29	0.02	138.70
7	4.32	116.69	13	0.98	83.61	19	0.42	96.46	25	0.23	118.51	31	0.15	142.15
7	3.98	116.14	13	0.90	83.77	19	0.39	96.13	25	0.21	117.89	31	0.14	139.66
7	3.64	115.13	13	0.83	84.02	19	0.36	96.19	25	0.20	117.16	31	0.12	140.05
7	3.30	113.89	13	0.75	84.48	19	0.32	97.07	25	0.18	116.31	31	0.11	140.51
7	2.96	112.45	13	0.67	85.03	19	0.29	98.08	25	0.16	117.78	31	0.10	141.02
7	2.62	110.95	13	0.59	85.68	19	0.26	98.82	25	0.14	119.24	31	0.09	141.35
7	2.28	109.19	13	0.52	86.56	19	0.23	100.00	25	0.13	120.44	31	0.08	140.02
7	1.94	107.31	13	0.44	87.49	19	0.20	101.29	25	0.11	121.01	31	0.07	139.29
7	1.59	105.50	13	0.36	88.98	19	0.16	102.51	25	0.09	120.18	31	0.05	141.02
7	1.25	103.92	13	0.28	90.03	19	0.13	103.16	25	0.08	121.50	31	0.04	142.76
7	0.91	102.60	13	0.21	91.53	19	0.10	104.59	25	0.06	122.78	31	0.03	144.52
7	0.57	101.69	13	0.13	93.03	19	0.07	105.53	25	0.04	124.09	31	0.02	146.30
7	0.23	101.49	13	0.05	93.60	19	0.04	106.54	25	0.02	125.43	31	0.01	148.09

Table 2: Percent Difference in Expected Dual-Command Travel and Areas for Fishbone and Traditional Warehouse Designs

	$E[DC]$					Area				
T	Fishbone	Layout A	%diff	Layout B	%diff	Fishbone	Layout A	%diff	Layout B	%diff
50	37.8	41.4	8.8%	42.7	11.5%	520.6	400.0	30.2%	475.0	9.6%
100	51.5	57.3	10.1%	56.7	9.2%	863.9	710.0	21.7%	725.0	19.2%
150	61.4	68.8	10.8%	67.5	9.0%	1192.2	960.0	24.2%	1065.0	11.9%
200	69.5	78.9	11.9%	77.1	9.9%	1501.0	1270.0	18.2%	1405.0	6.8%
250	77.4	88.0	12.0%	85.0	8.9%	1808.1	1520.0	19.0%	1655.0	9.2%
300	83.6	95.8	12.7%	92.7	9.9%	2103.9	1830.0	15.0%	1995.0	5.5%
350	90.0	103.2	12.8%	99.2	9.2%	2404.1	2080.0	15.6%	2245.0	7.1%
400	95.7	110.1	13.1%	105.6	9.3%	2692.5	2390.0	12.7%	2495.0	7.9%
450	100.9	116.4	13.3%	111.5	9.6%	2977.8	2640.0	12.8%	2835.0	5.0%
500	106.3	122.7	13.4%	117.0	9.1%	3268.2	2890.0	13.1%	3085.0	5.9%
550	110.8	128.3	13.6%	122.4	9.5%	3555.2	3200.0	11.1%	3335.0	6.6%
600	115.4	133.8	13.8%	127.6	9.5%	3835.0	3450.0	11.2%	3675.0	4.4%
650	120.0	139.3	13.8%	132.3	9.3%	4119.5	3700.0	11.3%	3925.0	5.0%
700	124.1	144.2	13.9%	137.0	9.4%	4403.6	4010.0	9.8%	4175.0	5.5%
750	128.1	149.1	14.1%	141.7	9.6%	4677.6	4260.0	9.8%	4425.0	5.7%
800	132.2	153.9	14.1%	145.9	9.4%	4962.5	4510.0	10.0%	4765.0	4.1%
850	136.2	158.6	14.1%	150.1	9.3%	5238.6	4820.0	8.7%	5015.0	4.5%
900	139.6	162.9	14.3%	154.2	9.5%	5514.7	5070.0	8.8%	5265.0	4.7%
950	143.3	167.2	14.3%	158.4	9.6%	5790.1	5320.0	8.8%	5515.0	5.0%
1000	146.9	171.6	14.4%	162.2	9.4%	6068.8	5570.0	9.0%	5855.0	3.7%
1050	150.5	175.7	14.4%	165.9	9.3%	6344.3	5880.0	7.9%	6105.0	3.9%
1100	153.5	179.6	14.5%	169.6	9.5%	6617.2	6130.0	7.9%	6355.0	4.1%
1150	156.8	183.6	14.6%	173.3	9.5%	6889.8	6380.0	8.0%	6605.0	4.3%
1200	160.2	187.5	14.6%	177.0	9.5%	7167.8	6630.0	8.1%	6945.0	3.2%
1250	163.3	191.3	14.6%	180.3	9.4%	7438.4	6940.0	7.2%	7195.0	3.4%
1300	166.4	194.9	14.6%	183.7	9.4%	7714.3	7190.0	7.3%	7445.0	3.6%
1350	169.2	198.5	14.7%	187.0	9.5%	7980.8	7440.0	7.3%	7695.0	3.7%
1400	172.3	202.1	14.7%	190.4	9.5%	8256.5	7690.0	7.4%	7945.0	3.9%
1450	175.3	205.7	14.8%	193.7	9.5%	8526.8	7940.0	7.4%	8285.0	2.9%
1500	178.2	209.1	14.8%	196.7	9.4%	8798.8	8250.0	6.7%	8535.0	3.1%
1550	180.9	212.4	14.8%	199.8	9.5%	9069.6	8500.0	6.7%	8785.0	3.2%
1600	183.6	215.7	14.9%	202.9	9.5%	9336.3	8750.0	6.7%	9035.0	3.3%
1650	186.4	219.0	14.9%	206.0	9.5%	9610.9	9000.0	6.8%	9285.0	3.5%
1700	189.1	222.3	14.9%	209.0	9.5%	9878.8	9250.0	6.8%	9535.0	3.6%
1750	191.8	225.5	14.9%	211.9	9.5%	10148.7	9560.0	6.2%	9875.0	2.8%
1800	194.4	228.5	14.9%	214.7	9.5%	10420.0	9810.0	6.2%	10125.0	2.9%
1850	196.8	231.6	15.0%	217.5	9.5%	10684.0	10060.0	6.2%	10375.0	3.0%
1900	199.4	234.6	15.0%	220.4	9.5%	10954.8	10310.0	6.3%	10625.0	3.1%
1950	202.0	237.7	15.0%	223.2	9.5%	11225.5	10560.0	6.3%	10875.0	3.2%
2000	204.4	240.7	15.1%	226.0	9.5%	11491.1	10870.0	5.7%	11215.0	2.5%
2050	206.9	243.6	15.1%	228.6	9.5%	11762.9	11120.0	5.8%	11465.0	2.6%
2100	209.2	246.4	15.1%	231.2	9.5%	12027.3	11370.0	5.8%	11715.0	2.7%
2150	211.6	249.3	15.1%	233.9	9.5%	12292.7	11620.0	5.8%	11965.0	2.7%
2200	214.0	252.2	15.1%	236.5	9.5%	12563.9	11870.0	5.8%	12215.0	2.9%
2250	216.4	255.0	15.2%	239.1	9.5%	12830.4	12120.0	5.9%	12465.0	2.9%
2300	218.6	257.8	15.2%	241.7	9.5%	13096.0	12430.0	5.4%	12715.0	3.0%
2350	221.0	260.5	15.2%	244.2	9.5%	13366.9	12680.0	5.4%	13055.0	2.4%
2400	223.1	263.1	15.2%	246.6	9.5%	13629.8	12930.0	5.4%	13305.0	2.4%
2450	225.3	265.8	15.2%	249.1	9.5%	13894.1	13180.0	5.4%	13555.0	2.5%
2500	227.6	268.5	15.2%	251.5	9.5%	14163.4	13430.0	5.5%	13805.0	2.6%

T	*E* [*DC*]					Area				
	Fishbone	**Layout A**	**%diff**	**Layout B**	**%diff**	**Fishbone**	**Layout A**	**%diff**	**Layout B**	**%diff**
2550	229.8	271.1	15.2%	254.0	9.5%	14430.5	13680.0	5.5%	14055.0	2.7%
2600	231.9	273.8	15.3%	256.4	9.5%	14693.9	13990.0	5.0%	14305.0	2.7%
2650	234.1	276.3	15.3%	258.8	9.6%	14962.1	14240.0	5.1%	14555.0	2.8%
2700	236.3	278.8	15.3%	261.1	9.5%	15228.2	14490.0	5.1%	14895.0	2.2%
2750	238.3	281.3	15.3%	263.4	9.6%	15489.7	14740.0	5.1%	15145.0	2.3%
2800	240.4	283.8	15.3%	265.7	9.5%	15755.7	14990.0	5.1%	15395.0	2.3%
2850	242.5	286.3	15.3%	268.0	9.5%	16026.2	15240.0	5.2%	15645.0	2.4%
2900	244.5	288.9	15.3%	270.3	9.5%	16287.5	15490.0	5.1%	15895.0	2.5%
2950	246.5	291.3	15.4%	272.5	9.5%	16551.7	15800.0	4.8%	16145.0	2.5%
3000	248.6	293.7	15.4%	274.8	9.6%	16820.2	16050.0	4.8%	16395.0	2.6%
3050	250.6	296.0	15.4%	277.0	9.5%	17082.8	16300.0	4.8%	16735.0	2.1%
3100	252.5	298.4	15.4%	279.2	9.6%	17344.4	16550.0	4.8%	16985.0	2.1%
3150	254.5	300.8	15.4%	281.3	9.5%	17610.0	16800.0	4.8%	17235.0	2.2%
3200	256.5	303.1	15.4%	283.5	9.5%	17879.6	17050.0	4.9%	17485.0	2.3%
3250	258.4	305.5	15.4%	285.6	9.5%	18139.8	17300.0	4.9%	17735.0	2.3%
3300	260.3	307.8	15.4%	287.7	9.5%	18403.0	17610.0	4.5%	17985.0	2.3%
3350	262.2	310.1	15.4%	289.9	9.6%	18670.1	17860.0	4.5%	18235.0	2.4%
3400	264.1	312.3	15.4%	292.0	9.5%	18936.3	18110.0	4.6%	18485.0	2.4%
3450	265.9	314.5	15.5%	294.1	9.6%	19193.8	18360.0	4.5%	18825.0	2.0%
3500	267.8	316.8	15.5%	296.1	9.5%	19457.6	18610.0	4.6%	19075.0	2.0%
3550	269.7	319.0	15.4%	298.1	9.5%	19725.0	18860.0	4.6%	19325.0	2.1%
3600	271.6	321.3	15.5%	300.1	9.5%	19988.0	19110.0	4.6%	19575.0	2.1%
3650	273.3	323.5	15.5%	302.2	9.5%	20249.1	19360.0	4.6%	19825.0	2.1%
3700	275.1	325.6	15.5%	304.2	9.6%	20513.6	19670.0	4.3%	20075.0	2.2%
3750	276.9	327.8	15.5%	306.2	9.6%	20781.5	19920.0	4.3%	20325.0	2.2%
3800	278.7	329.9	15.5%	308.2	9.6%	21039.7	20170.0	4.3%	20575.0	2.3%
3850	280.5	332.0	15.5%	310.2	9.6%	21300.7	20420.0	4.3%	20915.0	1.8%
3900	282.3	334.1	15.5%	312.1	9.5%	21564.9	20670.0	4.3%	21165.0	1.9%
3950	284.1	336.3	15.5%	314.0	9.5%	21832.4	20920.0	4.4%	21415.0	1.9%
4000	285.8	338.4	15.5%	315.9	9.5%	22092.3	21170.0	4.4%	21665.0	2.0%
4050	287.5	340.5	15.6%	317.8	9.6%	22353.4	21480.0	4.1%	21915.0	2.0%
4100	289.2	342.5	15.6%	319.7	9.6%	22617.6	21730.0	4.1%	22165.0	2.0%
4150	290.9	344.5	15.6%	321.7	9.6%	22884.8	21980.0	4.1%	22415.0	2.1%
4200	292.6	346.6	15.6%	323.6	9.6%	23142.2	22230.0	4.1%	22665.0	2.1%
4250	294.3	348.6	15.6%	325.5	9.6%	23402.5	22480.0	4.1%	22915.0	2.1%
4300	296.0	350.6	15.6%	327.3	9.6%	23665.9	22730.0	4.1%	23255.0	1.8%
4350	297.7	352.6	15.6%	329.1	9.5%	23932.1	22980.0	4.1%	23505.0	1.8%
4400	299.4	354.6	15.6%	330.9	9.5%	24193.0	23230.0	4.1%	23755.0	1.8%
4450	301.0	356.7	15.6%	332.8	9.6%	24452.9	23480.0	4.1%	24005.0	1.9%
4500	302.6	358.6	15.6%	334.6	9.6%	24715.6	23790.0	3.9%	24255.0	1.9%