

# A Unit-Load Warehouse with Multiple Pickup & Deposit Points and Non-Traditional Aisles

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## **Abstract**

We show how to configure a cross aisle in a unit-load warehouse to facilitate travel between storage locations and multiple pickup and deposit points on one side. We use our models to investigate designs having two types of cross aisles—those that form a “Flying-V” and those that form an “Inverted-V.” Our numerical results suggest that there is a benefit to using a Flying-V aisle design, but the benefit is more modest than in the case of a single P&D point. Thus, to the extent practicable, pickup and deposit points should be concentrated toward the middle of the warehouse.

[The Appendix at the end of this manuscript is intended as an online supplement.]

# 1 Introduction

The role of worldwide logistics continues to grow as manufacturers and markets become more distributed (Bartholdi et al., 2008). An important node in these logistics systems is the distribution center (or warehouse), where incoming goods are held until the final destination of the goods is known. With more than 500,000 warehouses in the U.S. alone (Energy Information Administration, 2006), models that assist in their design have the potential for significant application. Recent survey papers by Gu et al. (2007, 2009) illustrate the breadth and depth of research in this area, with models covering both design and analysis of the many subsystems of a distribution center. However, until recently, a fundamental question about the design of the aisles used to facilitate flow within the warehouse was left unanswered.

In a warehouse, racks are traditionally arranged to form parallel picking aisles and perhaps one or more orthogonal cross aisles, which facilitate flow but contain no product. In a traditional unit-load warehouse, in which products are received, stored, and handled (typically) on pallets, there is no advantage to inserting a cross aisle because doing so effectively moves half of the product farther from the pickup and deposit point (Pohl et al., 2009a). Gue and Meller (2009) described non-traditional designs in which the “rules” requiring parallel picking aisles and orthogonal cross aisles were relaxed. They addressed the question of how to design an inserted cross aisle to facilitate flow in a unit-load warehouse from a single, centrally-located pickup & deposit (P&D) point, showing the expected distance between a central P&D point and a location in the warehouse can be reduced by inserting non-orthogonal cross aisles, and by orienting the picking aisles in different directions. They developed two designs from non-linear optimization models. The Flying-V design (see Figure 1), which contains a cross aisle projecting diagonally in a piecewise linear fashion from the P&D point, offers approximately a 10 percent reduction in expected distance to a random location for reasonably-sized warehouses. The Fishbone design (Figure 1) confers a reduction of about 20 percent. An early paper by White (1972) proposed “radial aisles” that are similar in spirit to the fishbone design.

Additional work in the area of non-traditional aisle design has modified some of the

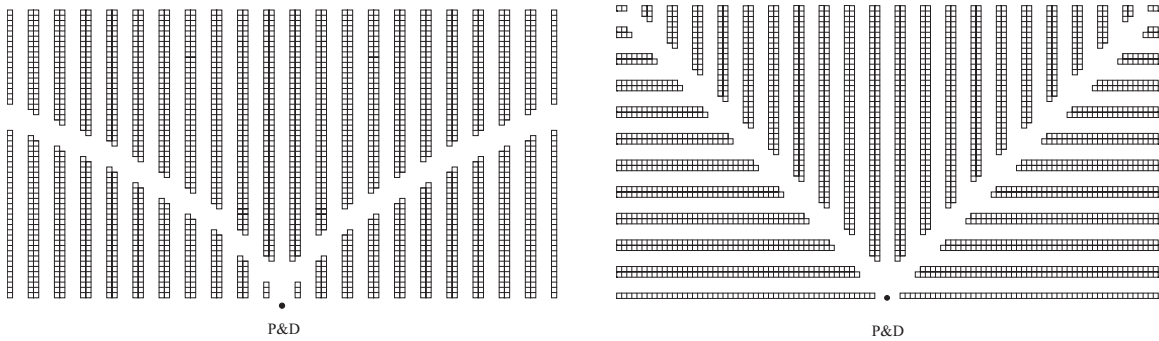


Figure 1: The Flying-V and Fishbone designs introduced by Gue and Meller (2009).

assumptions presented in Gue and Meller (2009). For example, Pohl et al. (2009b) consider the design of fishbone aisles when the operating protocol is dual-commands (sometimes called *task interleaving*), in which each trip into the warehouse space visits two locations—first a stow, then a retrieval, then back to the P&D point. They showed that for dual-command operations, the fishbone design is superior to a traditional design both with and without a (traditional) middle aisle, although the benefit is not quite as great as in the single-command case. For dual commands, the expected distance in a fishbone design is approximately 10–15 percent less than in a comparable, traditional design.

Pohl et al. (2009c) examine two non-traditional aisle designs and three traditional designs under dedicated (rather than random) storage policies and either single- or dual-command operations. Their work indicates that although the fishbone design performs well in all instances, the Flying-V does not perform well (i.e., it has difficulty outperforming traditional designs) when the relative activity of stored items is significantly different than uniform, and when it is operated under dual-command operations.

Prior research concerning warehouse design with multiple P&D points can be found in Francis et al. (1992) and other textbooks, where models based on a continuous warehouse space are used to design contour lines describing equivalent expected distances from the set of P&D points. Because these models assume a continuous space, aisles are not considered, other than imposing rectilinear travel in a conventional warehouse and Tchebychev travel in an AS/RS aisle. Multiple P&D points have also been examined in the context of an order

picking system by Eisenstein (2008), and more broadly in a manufacturing assembly facility by Yano et al. (1998). In an abbreviated version of the present paper, Gue et al. (2008) considered multiple P&D points in a warehouse.

Meller and Gue (2009) provide a description of two early implementations of non-traditional aisle designs in practice. In the first, a “laydown area” is used to transfer loads from the material handling device for removing loads from trailers to the material handling device for storing loads (and vice versa). Such a laydown area is a good example of single P&D point. In the second implementation, multiple P&D points were used on the receiving side of the warehouse and the non-traditional aisle design did not offer much benefit. The positive benefits from the aisle design were on the opposite (shipping) side, where each pallet passed through a single stretch-wrapping machine. In both cases, managers and operators have been happy with the designs and have reported no major difficulties.

Our review of existing research indicates that the fishbone design is generally preferred to the Flying-V, especially for new construction (versus retrofitting an existing facility). However, existing aisle design papers assume that travel begins and ends at a single P&D point. For many operations, of course, this is not a good model of travel. For example, the shipping area in a warehouse may have two, three, or even more stretch-wrap machines, meaning that travel begins and ends from two or more points. In other applications, an entire side of the warehouse space may contain P&D points, as in the case of a shipping area with dozens of shipping doors. Should such warehouses be designed with a non-orthogonal cross aisle? If so, what shape should it be and how much benefit is possible? We address these questions in this paper.

As in Gue and Meller (2009), we are considering a unit-load warehouse, in which pallets are received and stored until ordered, when they are moved to shipping without being broken down into cases. In many warehouses, pallet movement is accomplished with a powered, pallet-based material handling vehicle (pallet truck, counter-balanced lift truck, reach trucks, or narrow-aisle straddle trucks). However, movement could also be accomplished with a pallet jack, or in some cases, carts, for unit loads smaller than a pallet. The operating policy

of the facility is single-command operations, in which every trip begins and ends at a P&D location and visits a single storage location. For workload balance reasons, the worker may serve more than one P&D point.

The requirement for the warehouse is to unload and deliver all loads from P&D points to their storage locations (receiving) and to retrieve requested loads and take them to their appropriate P&D points (shipping). We assume random storage and uniform usage of P&D points, which leads to the objective to minimize the expected distance from a random dock door to a random storage location. (Note that this objective is equivalent to minimizing the expected distance from a random storage location to a random dock door.)

In practice, unit-load warehouses using single-command cycles use a variety of worker assignment policies. In the simplest policy, workers are assigned exclusively to particular doors and therefore make all their trips to the storage area from the same P&D point. Our objective function models this policy directly. More complex policies attempt to reduce empty travel by directing workers to nearby picks or doors after they have delivered their unit loads. Although our objective function does not model these policies directly, it does facilitate them by reducing expected distances between storage locations and P&D points in general. We contend that opportunistic routing policies would tend to have the same relative effect on any design, and therefore that a good design for operations without intelligent routing will be a good design for operations with it as well.

Because the fishbone design has a clear orientation toward a single P&D point, we do not consider it further here. For a symmetric warehouse with parallel picking aisles, the cross aisle in one half is either non-monotonic, monotonically increasing (as in the Flying-V), or monotonically decreasing. The non-monotonic case is difficult to model and, in our judgment, not very promising. In the next section we treat the monotonically increasing case by introducing a model of travel in a unit-load warehouse with multiple P&D points. We use that model to determine the best shape for such a cross aisle, which we call a “modified Flying-V,” assuming travel may begin and end at any of the multiple P&D points. In Section 3 we treat the monotonically decreasing case, which results in an inserted cross aisle

shaped as an “Inverted V.” In Section 4, we compare the two designs and show that the modified Flying-V design is superior, and that the benefits from the modified Flying-V cross aisle decrease as the number of P&D points increases. We summarize the paper in the final section.

## 2 Travel in the modified Flying-V warehouse

In the previous section we used the terms “Flying-V” and “modified Flying-V” to suggest a difference between designs for one or multiple P&D points, but in the remaining sections we drop the term “modified,” for ease of exposition.

### 2.1 Assumptions

As in Gue and Meller (2009), we model the warehouse as a discrete set of picking aisles, each containing a continuous, uniform distribution of picking activity. These assumptions are intended to represent a random storage policy (or equivalently, the closest-open-location policy in the case of a fully-utilized warehouse), which is common in unit-load warehouses (see Gue and Meller, 2009). We assume travel begins with equal probability from any of  $n$  P&D points at the bottom of the space, where  $n$  is also the number of picking aisles. That is, we consider an extreme case in which the number of P&D points is equal to the number of picking aisles. (We argue in the conclusions that this assumption effectively produces a worst-case analysis with respect to expected benefit of an inserted cross aisle.) We assume that the warehouse is symmetric about a vertical axis passing through its center.

We also assume, implicitly, a prescribed *routing rule* for workers (see Figure 2). For example, assume we stand at a P&D point  $i$  in front of picking aisle  $i$  in the right half of the warehouse. The routing rule is

- for a location in the right half of the warehouse,
  - if the location is in aisle  $i$ , travel directly to it (path not shown in Figure 2).

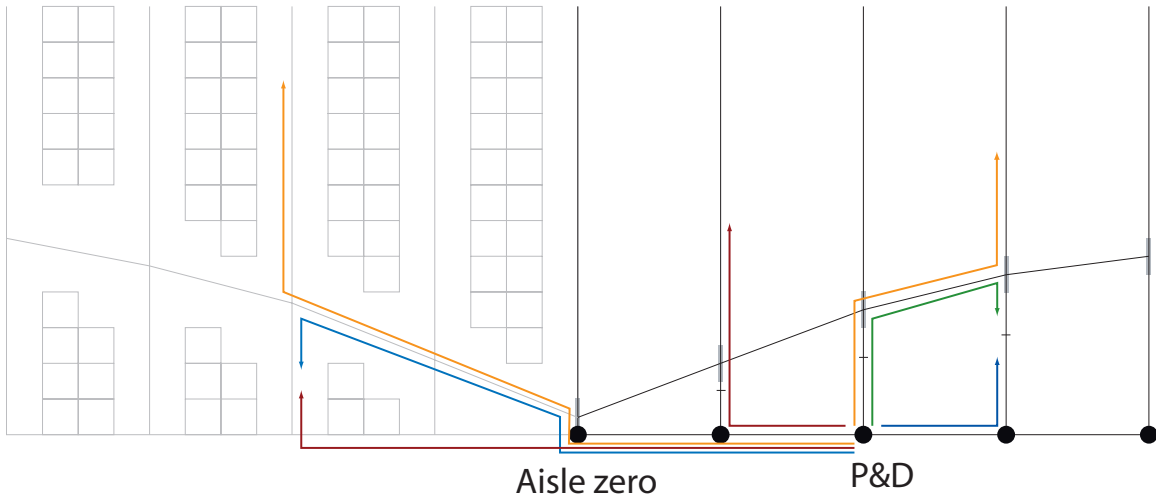


Figure 2: Flying-V shape — scheme of travel patterns and related picking regions.

- if the location is to the right and above the cross aisle, travel up aisle  $i$ , then along the cross aisle, then up the destination aisle.
- if the location is to the right and just below the cross aisle (we clarify this below), travel up aisle  $i$ , then along the cross aisle, then down the destination aisle.
- if the location is to the right and near the bottom of the aisle, travel right along the bottom cross aisle, and then up the destination aisle.
- if the location is to the left (but still in the right half of the warehouse), travel left along the bottom cross aisle, then up the destination aisle.
- for a location in the left half of the warehouse,
  - travel along the bottom cross aisle to the center picking aisle, then follow the rules above in “mirror” fashion.

Observe that the routing rule imposes a particular structure on the middle cross aisle, although not its precise specification: the rule is sensible only if the middle aisle has positive slope in the right half of the warehouse and negative in the left. This compromise is necessary to resolve a modeling conundrum: routing of workers depends on the structure of the aisles, and the structure of the aisles depends on the routing of workers. Ideally, we would search

the space of aisle designs and evaluate each one by solving shortest path problems, but we deemed this approach intractable. Instead, a defined routing rule yields a closed-form expression for the expected distance to a location in the warehouse, which we can easily embed in a non-linear optimization model.

Our modeling approach follows that described in Gue and Meller (2009), although the models we describe here are more general because we account for multiple P&D points and more potential travel paths. Assume there are  $n$  picking aisles and the warehouse is  $h$  units high, where a unit is “one pallet length” (about 4 feet). The distance between picking aisle centers is  $a$ . Because we assume the cross aisle is symmetric about the center aisle, we consider only the right half of the warehouse. We number picking aisles in the right half-space starting from the center and ending at the rightmost aisle. Thus,  $i = 0, \dots, m$ , where  $m = (n - 1)/2$ . We describe the middle cross aisle with a vector  $\vec{b} = \{b_0, b_1, \dots, b_m\}$ , where  $b_i$  is the point of intersection of the middle aisle with picking aisle  $i$ . As in Gue and Meller (2009), we assume the cross aisle consumes  $2w$  of a picking aisle at points of intersection with picking aisles. Note that all P&D points should be moved “down” a distance equal to the width of half a horizontal cross aisle to facilitate travel to the left or right in the bottom cross aisle. Because this distance is constant for all designs, we ignore it.

## 2.2 The model

We divide the picking space according to appropriate travel patterns (see Figure 2), and define the following notation:

- $E[R_i]$  is the expected distance to a location to the *right* of aisle  $i$ ;
- $E[L_i]$  is the expected distance to a location to the *left* of aisle  $i$ , but no farther left than the center picking aisle;
- $E[M_i]$  is the expected distance to a location to the left of aisle  $i$  and beyond the center aisle, which we call the *leftmost* region; and



- $E[S_i]$  is the expected distance to a location in aisle  $i$ .

The expected distance to a random location from a randomly chosen P&D point  $i$  in the right half of the warehouse is

$$E[C_i] = p_{R_i}E[R_i] + p_{S_i}E[S_i] + p_{L_i}E[L_i] + p_{M_i}E[M_i],$$

where  $p_{R_i} = (m - i)/n$ ,  $p_{S_i} = 1/n$ ,  $p_{L_i} = i/n$ , and  $p_{M_i} = m/n$  are the probabilities of visiting each of the respective regions. These probabilities are valid only for the case we consider here, in which each picking aisle has a corresponding P&D point and  $n = 2m + 1$ . An appropriate modification is easily obtained for cases with fewer P&D points.

Due to symmetry, the total expected distance from an arbitrary P&D point is

$$E[C] = p_0E[C_0] + 2 \sum_{i=1}^m p_i E[C_i], \quad (1)$$

where  $p_j = 1/n$  is the probability of choosing P&D point  $j$ .

The expression for  $E[C]$  requires estimated distances for each of the travel patterns in Figure 2. Consider aisles  $i$  and  $r$ , such that  $i < r$ , and assume that a pick is being made from P&D point  $i$  to a random location in aisle  $r$ . The worker should choose a route based on the how far into aisle  $r$  the location is. If the location is above the cross aisle, the worker travels straight ahead in aisle  $i$ , along the cross aisle, and then up aisle  $r$  — the upper path. If the location is between the middle aisle and the bottom cross aisle, there are two possible routes — along the cross aisle and down, or along the bottom aisle and up. There exists, then, a point  $q_{ir}$  in aisle  $r$  at which the worker is indifferent between the paths if he is traveling from P&D point  $i$ .

The middle aisle has length  $d_j = \sqrt{a^2 + (b_j - b_{j-1})^2}$  between aisles  $j$  and  $j - 1$ , and the distance between aisles  $i$  and  $r$  along the middle aisle is  $d_{ir} = \sum_{j=i+1}^r d_j$ . The point of indifference  $q_{ir}$  in aisle  $r$  for a worker traveling from P&D point  $i$  is determined by setting

the two path lengths equal to one another,

$$\begin{aligned}(r-i)a + q_{ir} &= b_i + d_{ir} + b_r - q_{ir} \\ q_{ir} &= \frac{1}{2} \left[ b_i + d_{ir} + b_r - (r-i)a \right].\end{aligned}$$

For the picks above  $q_{ir}$ , the path along the middle aisle is preferred and for those below, the lower path is best.

Before developing expressions for the expected distance for each travel pattern, we must discuss and resolve an anomaly of the model. Because the middle aisle consumes  $2w$  of each picking aisle, we impose the constraint  $w \leq b_r \leq h - w$ . For logical reasons, we require  $w \leq q_{ir} \leq b_r - w$ , which is to say, the point of indifference in a picking aisle must be in a valid position within the aisle, otherwise it has no meaning. Therefore, for  $i < r$ ,

$$\begin{aligned}q_{ir} &\leq b_r - w \\ \frac{1}{2}(b_i + d_{ir} + b_r - (r-i)a) &\leq b_r - w \\ d_{ir} - (b_r - b_i) &\leq (r-i)a - 2w \\ \sum_{j=i+1}^r \sqrt{a^2 + (b_j - b_{j-1})^2} - (b_r - b_i) &\leq (r-i)a - 2w.\end{aligned}$$

Adjacent aisles are especially sensitive to this constraint. In this case,

$$\sqrt{a^2 + (b_j - b_{j-1})^2} - (b_j - b_{j-1}) \leq a - 2w,$$

which simplifies to

$$b_j - b_{j-1} \geq \frac{2w(a-w)}{a-2w}.$$

This inequality effectively imposes a lower bound on the slope of consecutive cross aisle segments. It tends to be tight when  $h$  is small compared to the warehouse width  $A = na$ . In extreme cases, the constraint produces a straight V-shaped middle aisle with the rightmost

$b_i$ 's either being very close to, or reaching  $h - w$ . If the latter occurs; i.e.,  $b_r = h - w$  and  $r < m$ , all aisles with indices greater than  $r$  do not have a cross aisle.

It is important to note that these constraints are not physical, but rather artifacts of the model. If the constraint is removed, a cross aisle with a very low slope could cause our expression for  $E[R_i]$  to become negative. To do away with the need for such a constraint, we define  $\mathbf{q}_{ir} = \min(q_{ir}, b_r - w)$ , which ensures that a point of indifference is in a valid position.

The expected distance to a location to the right of P&D point  $i$  is

$$\begin{aligned}
E[R_i] &= \sum_{r=i+1}^m \frac{1}{(m-i)(h-2w)} \left[ \int_0^{\mathbf{q}_{ir}} [(r-i)a+x]dx \right. \\
&\quad \left. + \int_{\mathbf{q}_{ir}}^{b_r-w} [b_i+d_{ir}+b_r-x]dx + \int_{b_r+w}^h [b_i+d_{ir}+x-b_r]dx \right] \\
&= \sum_{r=i+1}^m \frac{1}{(m-i)(h-2w)} \left[ \mathbf{q}_{ir} \left[ (r-i)a + \frac{\mathbf{q}_{ir}}{2} \right] + (b_r-w-\mathbf{q}_{ir}) \left[ b_i+d_{ir} \right. \right. \\
&\quad \left. \left. + \frac{1}{2}(b_r+w-\mathbf{q}_{ir}) \right] + (h-b_r-w) \left[ b_i+d_{ir} + \frac{1}{2}(h-b_r+w) \right] \right].
\end{aligned}$$

The three main terms in this expression are, for each picking aisle to the right, the expected distance to a pick using the bottom cross aisle and up the picking aisle, using the diagonal aisle and down, and using the diagonal aisle and up, respectively. For a location to the left but not beyond the center aisle,

$$\begin{aligned}
E[L_i] &= \sum_{l=0}^{i-1} \frac{1}{i(h-2w)} \left[ \int_0^h [(i-l)a+x]dx - \int_{b_l-w}^{b_l+w} [(i-l)a+x]dx \right] \\
&= \sum_{l=0}^{i-1} \frac{1}{i(h-2w)} \left[ a(i-l)(h-2w) + \frac{h^2}{2} - 2b_lw \right].
\end{aligned}$$

Terms in this expression reflect, for each potential picking aisle, the distance to that aisle along the bottom aisle and the expected distance in the picking aisle. For a location in the

leftmost region,

$$\begin{aligned}
E[M_i] &= \sum_{\ell=1}^m \frac{1}{m(h-2w)} \left[ \int_0^{\mathbf{q}_{0\ell}} [ia + \ell a + x] dx \right. \\
&\quad + \int_{\mathbf{q}_{0\ell}}^{b_\ell-w} [ia + b_0 + d_{0\ell} + b_\ell - x] dx \\
&\quad \left. + \int_{b_\ell+w}^h [ia + b_0 + d_{0\ell} + x - b_\ell] dx \right] \\
&= \sum_{\ell=1}^m \frac{1}{m(h-2w)} \left[ \mathbf{q}_{0\ell} \left[ (i+\ell)a + \frac{\mathbf{q}_{0\ell}}{2} \right] + (b_\ell - w - \mathbf{q}_{0\ell}) [ia + b_0 + d_{0\ell} \right. \\
&\quad \left. + \frac{1}{2}(b_\ell + w - \mathbf{q}_{0\ell}) \right] + (h - b_\ell - w) \left[ ia + b_0 + d_{0\ell} + \frac{1}{2}(h - b_\ell + w) \right] \Big].
\end{aligned}$$

Terms in this expression are analogous to those for a pick to the right of the P&D point, except we must add the distance to the center aisle. The expected travel distance to a pick in the same aisle as the P&D point is

$$\begin{aligned}
E[S_i] &= \frac{1}{(h-2w)} \left[ \int_0^h x dx - \int_{b_i-w}^{b_i+w} x dx \right] \\
&= \frac{1}{(h-2w)} \left[ \frac{h^2}{2} - 2b_i w \right].
\end{aligned}$$

Finally, for the central picking aisle: Because expected travel distances to the right and left from the central aisle are the same, we can simplify the expression to:

$$E[C_0] = 2p_{R_0}E[R_0] + p_{S_0}E[S_0],$$

where  $p_{R_0} = m/n$  and  $p_0 = 1/n$  are the probabilities of picking to the right of aisle 0 and the probability of picking in aisle 0.

$$\begin{aligned}
E[R_0] &= \sum_{r=1}^m \frac{1}{m(h-2w)} \left[ \int_0^{\mathbf{q}_{0r}} [ra+x]dx \right. \\
&\quad \left. + \int_{\mathbf{q}_{0r}}^{b_r-w} [b_0+d_{0r}+b_r-x]dx + \int_{b_r+w}^h [b_0+d_{0r}+x-b_r]dx \right] \\
&= \sum_{r=1}^m \frac{1}{m(h-2w)} \left[ \mathbf{q}_{0r} \left( ra + \frac{\mathbf{q}_{0r}}{2} \right) \right. \\
&\quad \left. + (b_r-w-\mathbf{q}_{0r}) \left[ b_0+d_{0r} + \frac{1}{2}(b_r+w-\mathbf{q}_{0r}) \right] \right. \\
&\quad \left. + (h-b_r-w) \left[ b_0+d_{0r} + \frac{1}{2}(h-b_r+w) \right] \right].
\end{aligned}$$

And,

$$\begin{aligned}
E[S_0] &= \frac{1}{(h-2w)} \left[ \int_0^h xdx - \int_{b_0-w}^{b_0+w} xdx \right] \\
&= \frac{1}{(h-2w)} \left[ \frac{h^2}{2} - 2b_0w \right].
\end{aligned}$$

To summarize, we have developed expected distance expressions for each travel path from a P&D point to a destination location, and we have combined them into a single, non-linear expected distance expression, (1). The optimization problem is to

$$\text{Minimize } E[C]$$

subject to

$$w \leq b_i \leq h-w, i = 0, \dots, m.$$

The model solves in reasonable time with any of the search routines in MATHEMATICA. Solutions among those routines were essentially the same. Although it is possible that we are stuck in a local minimum for any of our solutions, we believe that solutions we describe

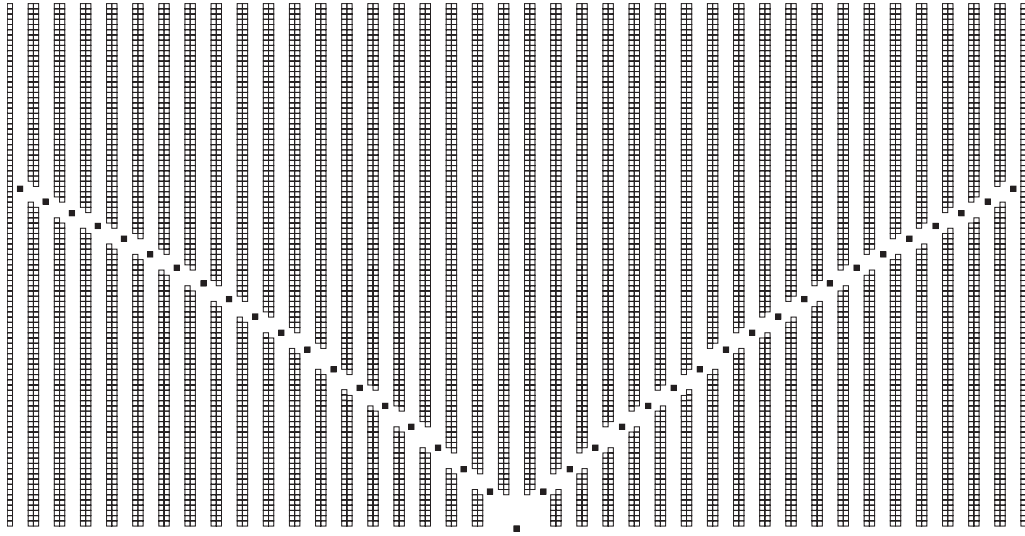


Figure 3: An example Flying-V design, with solid squares indicating  $b_i$  values.

here are optimal (or near-optimal) for three reasons: (1) The main structure of these solutions (a Flying-V) has been consistent among dozens of problems we have solved. (2) Solutions are relatively insensitive to changes in the exact shape of the cross aisle. For example, solutions that force the cross aisle not to have any curve at all (unlike the slight curve seen in Figure 3), are within 0.1 percent of the solution produced by our model. This is a mild argument for smoothness of the search space. (3) Solutions are in keeping with intuition.

The Flying-V design depicted in Figure 3 looks very much like solutions for the single P&D point cases shown in Gue and Meller (2009). The strong similarity between this design and the optimal design for a single P&D point (they are almost indistinguishable) suggests that the Flying-V is robust with respect to the number of P&D points. The middle aisle in Figure 3 has  $b_0 = w$ , which is as low as possible. This is beneficial for all travel paths that begin on one side of the warehouse and end on the other; the routing rule in this case states that, once at the center aisle, the middle aisle may be used. The middle aisle extends monotonically into the right and left halves of the warehouse, as we would expect. It is important to note, however, that this characteristic of our solutions is not required by constraint, but rather is a manifestation of the routing rules for travel. The model is free to

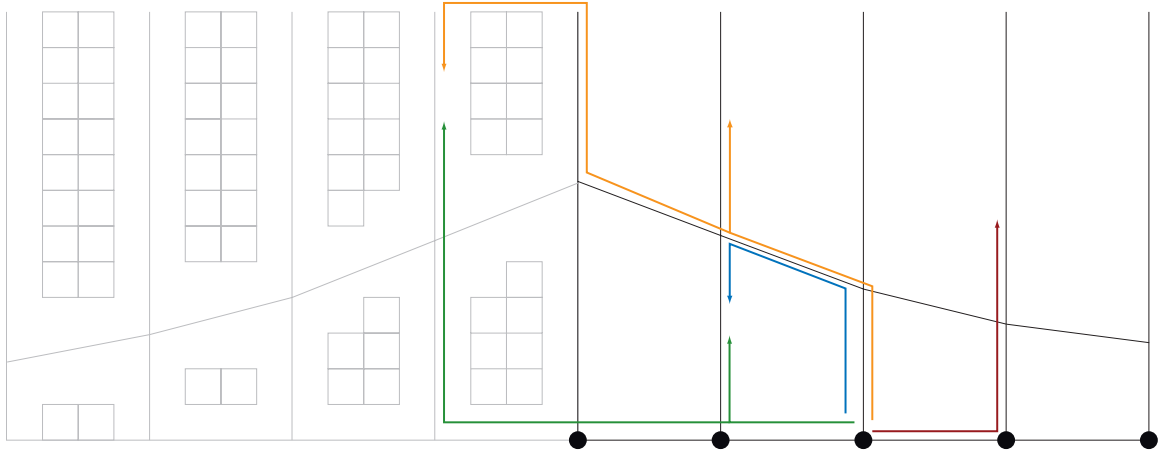


Figure 4: Travel patterns in the Inverted-V middle aisle.

choose any shape for the middle cross aisle.

Before we present further results for the Flying-V warehouse, we cover a model for warehouses with a middle cross aisle of a different shape.

### 3 Travel in the Inverted-V warehouse

Assumptions for this model are the same as for the Flying-V, and the development follows along similar lines. However, as Figure 4 illustrates, the travel paths are more numerous and slightly more complicated.

The routing rules that produce an Inverted-V middle aisle are:

- for a location in the right half of the warehouse,
  - if the location is in aisle  $i$ , travel directly to it.
  - if the location is to the left (but still in the right half of the warehouse) and above the cross aisle, travel up aisle  $i$ , then along the cross aisle, then up the destination aisle.
  - if the location is to the left and just below the cross aisle, travel up aisle  $i$ , then along the cross aisle, then down the destination aisle.

- if the location is to the left and near the bottom of the aisle, travel left along the bottom cross aisle, and then up the destination aisle.
- if the location is to the right, travel right along the bottom cross aisle, then up the destination aisle.
- for a location in the left half of the warehouse,
  - if the location is just below the top of the warehouse, travel up aisle  $i$ , left along the middle aisle to aisle 0, then to the top of the warehouse in aisle 0, then left along the top cross aisle to the destination aisle and down (see Figure 4).
  - if the location is just above the bottom of the warehouse, travel left along the bottom aisle to the destination aisle and up.
  - if the location is near the middle of the destination aisle, choose the shorter of two paths:
    - \* Travel up aisle  $i$ , left along the middle aisle to the destination aisle, then up or down as appropriate.
    - \* Travel left along the bottom aisle, then up the destination aisle.

These routing rules produce a middle aisle in the shape of an Inverted-V, although such a shape is not imposed by constraints in the model.

For a random P&D point in the right half of the warehouse, travel to the right in an Inverted-V design is equivalent to travel to the left in the Flying-V. The same holds for travel to the left (no farther than the central aisle) in the Inverted-V design and travel to the right in a Flying-V.

The expected distance expression for P&D point  $i$  is

$$E[C_i] = p_{R_i}E[R_i] + p_{S_i}E[S_i] + p_{L_i}E[L_i] + p_{M_i}E[M_i].$$

Development of the first three components is as before; the fourth (for the leftmost region) is slightly more complicated. Development of the four components is presented in the



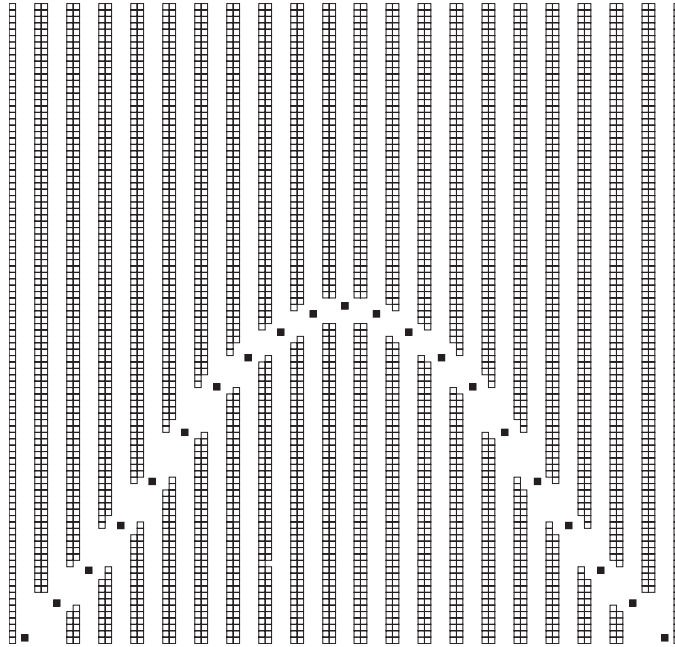


Figure 5: An example Inverted-V design, with solid squares indicating  $b_i$  values in the solution.

Appendix.

Again, our goal is to

$$\text{Minimize } E[C]$$

subject to

$$w \leq b_i \leq h - w, i = 0, \dots, m.$$

As before, the model solves in reasonable time with any of the search routines in MATHEMATICA, and solutions were almost the same among those routines. An example solution is provided in Figure 5.

The middle aisle in an Inverted-V design has a rounded shape to facilitate travel “over the top” and into the other side of the warehouse, but also to facilitate the travel patterns in Figure 8. Solutions for all configurations we considered had  $b_m = w$ , which brings the middle aisle as close as possible to the bottom of the warehouse near the ends.

## 4 A comparison of designs

Our objective is to discover whether or not alternative aisle designs can reduce expected distances in a unit-load warehouse with multiple P&D points, and if so, by how much.

We solved our models for a variety of configurations, with parameters

- number of aisles  $n = 11\text{--}39$  in increments of 4,
- width of the cross aisle  $w = 1\text{--}3$  in increments of 0.5, and
- height of the warehouse  $h = 50\text{--}125$  in increments of 25.

By way of reminder, units for  $w$ ,  $h$ , and  $a$  are “pallets lengths,” which is about 4 feet. We set  $a = 5$ , which implies that the distance between aisle centers is 20 feet, and therefore a picking aisle is 12 feet wide. Figure 6 shows the results for several sets of parameters.

For the Flying-V design we see that the performance (relative to a traditional warehouse) generally improves as the warehouse gets larger and for warehouses with more aisles. And, as expected, larger values of  $w$  degrade the possible improvement. In general, for warehouses of 15 or more aisles and  $h$  of 100 or more, the potential savings is approximately 3–6%. These potential savings do not compare favorably with those for warehouses with a single P&D point (see Gue and Meller, 2009), where we can expect the benefit of a Flying-V to be more than 10 percent in many cases.

For the Inverted-V design, the improvement over traditional designs improves with  $h$  and the number of aisles and decreases as  $w$  increases. However, the Inverted-V design does not offer any savings for warehouses less than about 15 aisles wide, and the savings are very small in any case. For the cases we tested, the Inverted-V design offered less than 2 percent savings.

The assumption that travel can begin or end at every point at the bottom of the warehouse is a kind of worst case problem because aisles must necessarily accommodate flows in many different directions. To illustrate this point, we changed the concentration of flows in a 31-aisle warehouse from the most concentrated case (a single P&D point) to the most diffused

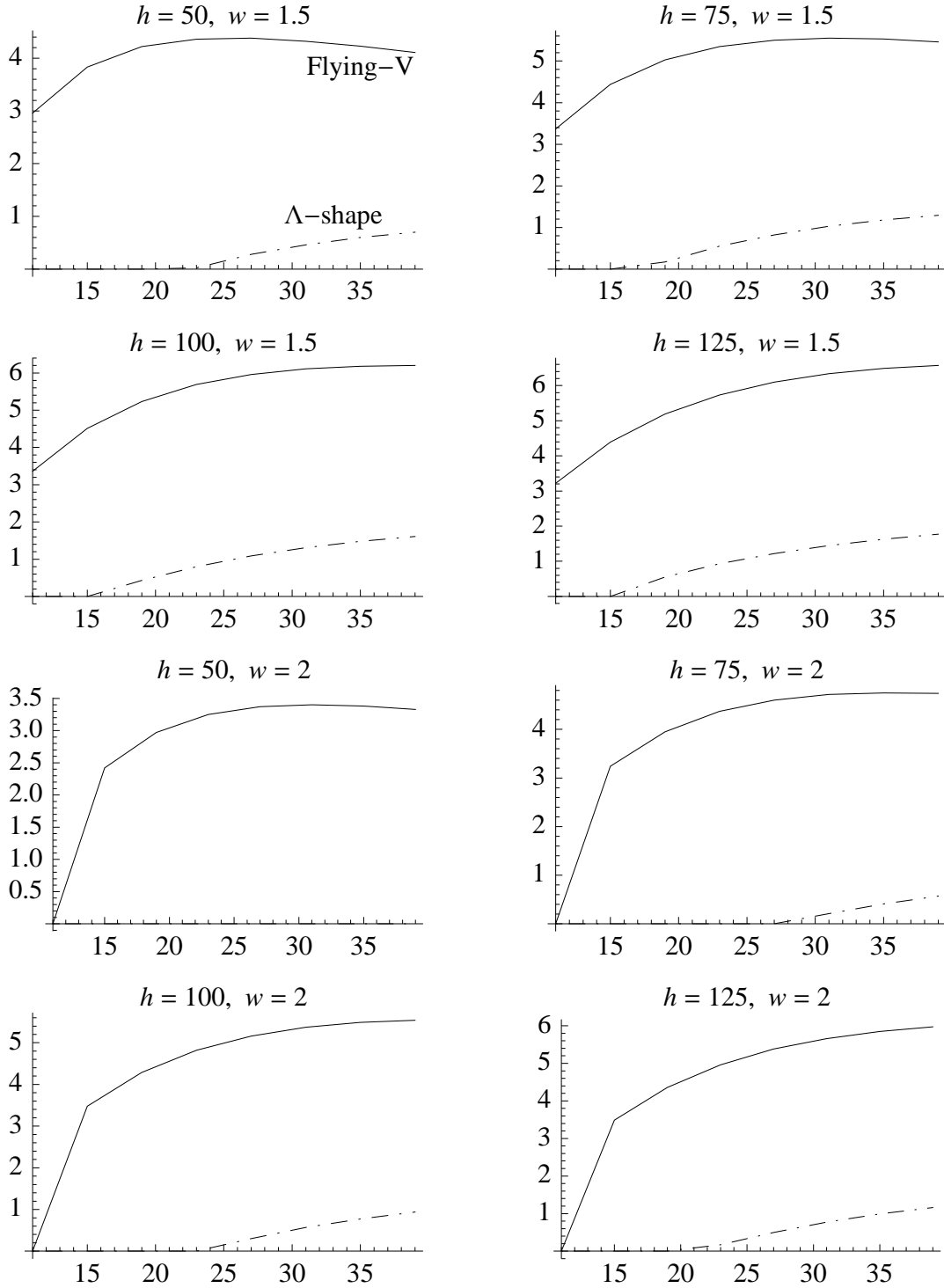


Figure 6: Percent savings for the Flying-V and Inverted-V designs as a function of the number of aisles over a traditional design with orthogonal travel.  $h$  is the length of the warehouse (in pallets);  $2w$  is approximately the width of the cross aisle.

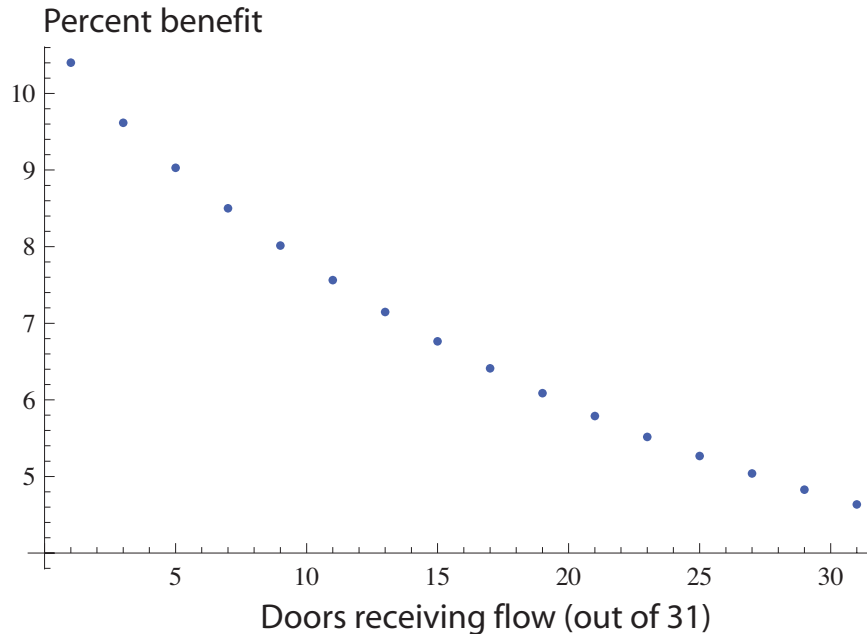


Figure 7: Benefit of an improved middle aisle for warehouses having different numbers of P&D points.

case (a P&D point at the end of every picking aisle). Figure 7 illustrates the improvement achieved by a Flying-V cross aisle as the number of doors increases. The most concentrated case has one door receiving flows, and there is approximately a 10 percent savings for the improved design, in keeping with the results of Gue and Meller (2009). As the number of doors receiving flow increases, the expected benefit decreases steadily, showing that more diffused flows along the bottom of the warehouse diminish the power of the middle aisle to facilitate efficient travel. Therefore, the results we have presented in this paper are a sort of worst-case scenario for flows.

## 5 Conclusions

Prior work in warehouse aisle design is limited by the assumption that there is a single P&D point. The contribution of our paper to the literature is the development of aisle designs, and models to configure those designs, that consider multiple P&D points.

We have proposed two new aisle designs, the modified Flying-V cross aisle and the Inverted-V cross aisle, with the goal of facilitating flow between multiple P&D points and locations in a unit-load warehouse. We investigated the performance of these designs for a variety of warehouse parameters (always assuming a positive width for the inserted cross aisle). We compared their performance individually to a traditional unit-load warehouse with no inserted cross aisle. This comparison illustrates that with the maximum number of P&D points,

*A modified Flying-V cross aisle can reduce travel 3–6% for reasonably-sized warehouses, when compared with a traditional warehouse.*

These results might seem disappointing when compared to the single P&D point case, where we can expect the benefit of a Flying-V to be more than 10 percent in many cases (see Gue and Meller, 2009). However, our testing indicated that the case of one P&D point for each aisle represents a worst case in terms of performance, because relative performance decreases as the number of P&D points increases. Therefore,

*The doors most often used should be those near the center, thus concentrating the material flows and improving the benefit of a Flying-V aisle. When there are fewer P&D points than doors, the P&D points should be located as close as possible to the center of the warehouse.*

The comparable result for the Inverted-V design is less than 1% for reasonably-sized warehouses, and negative improvement for some cases. We observed that,

*With the maximum number of P&D points, the modified Flying-V cross aisle warehouse always outperforms a comparable Inverted-V cross aisle.*

The decision of whether to implement a modified Flying-V cross aisle design centers on the tradeoff between productivity improvements and a lower storage density (due to the inserted cross aisle). The model and results in this paper will be helpful in establishing the former — at least to the extent that travel distances affect productivity — whereas we can

estimate the latter by noting that the modified Flying-V warehouse will be larger by a factor up to  $(h + 2w)/h$  (e.g., when  $h = 100$  pallet widths and  $w = 1.5$  pallet widths, the modified Flying-V warehouse is approximately 3% larger). There are some applications for which the inserted cross aisle could be implemented as a tunnel; in which case, the area increase is much smaller. Experience also suggests that the turns in a modified Flying-V warehouse can be executed at higher speed, which will positively affect productivity (Meller and Gue, 2009).

We conclude by noting that many more aisle designs are possible, especially those that do not require parallel picking aisles. Future research should investigate such designs, in addition to the impact on travel distances of non-random storage, task-interleaving, and the best placement of P&D points.

## Acknowledgement

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## Appendix (Intended as an Online Supplement)

Recall that the expected distance expression for P&D point  $i$  is

$$E[C_i] = p_{R_i}E[R_i] + p_{S_i}E[S_i] + p_{L_i}E[L_i] + p_{M_i}E[M_i].$$

Development of the first three components is as before; the fourth (for the leftmost region) is slightly more complicated. First,

$$\begin{aligned} E[R_i] &= \sum_{r=i+1}^m \frac{1}{(m-i)(h-2w)} \left[ \int_0^h [(r-i)a+x]dx - \int_{b_r-w}^{b_r+w} [(r-i)a+x]dx \right] \\ &= \sum_{r=i+1}^m \frac{1}{(m-i)(h-2w)} \left[ a(r-i)(h-2w) + \frac{h^2}{2} - 2b_r w \right], \\ E[S_i] &= \frac{1}{(h-2w)} \left[ \int_0^h xdx - \int_{b_i-w}^{b_i+w} xdx \right] \\ &= \frac{1}{(h-2w)} \left[ \frac{h^2}{2} - 2b_i w \right], \\ E[L_i] &= \sum_{l=0}^{i-1} \frac{1}{i(h-2w)} \left[ \int_0^{\mathbf{q}_{il}} [(i-l)a+x]dx + \int_{\mathbf{q}_{il}}^{b_l-w} [b_i+d_{il}+b_l-x]dx \right. \\ &\quad \left. + \int_{b_l+w}^h [b_i+d_{il}+x-b_l]dx \right] \\ &= \sum_{l=0}^{i-1} \frac{1}{i(h-2w)} \left[ \mathbf{q}_{il} \left( (i-l)a + \frac{\mathbf{q}_{il}}{2} \right) \right. \\ &\quad \left. + (b_l-w-\mathbf{q}_{il}) \left[ b_i+d_{il} + \frac{1}{2}(b_l+w-\mathbf{q}_{il}) \right] \right. \\ &\quad \left. + (h-b_l-w) \left[ b_i+d_{il} + \frac{1}{2}(h-b_l+w) \right] \right]. \end{aligned}$$

When traveling from the right half of the warehouse into the left, sometimes it is best to use the patterns in Figure 4 and sometimes the more complicated paths in Figure 8.



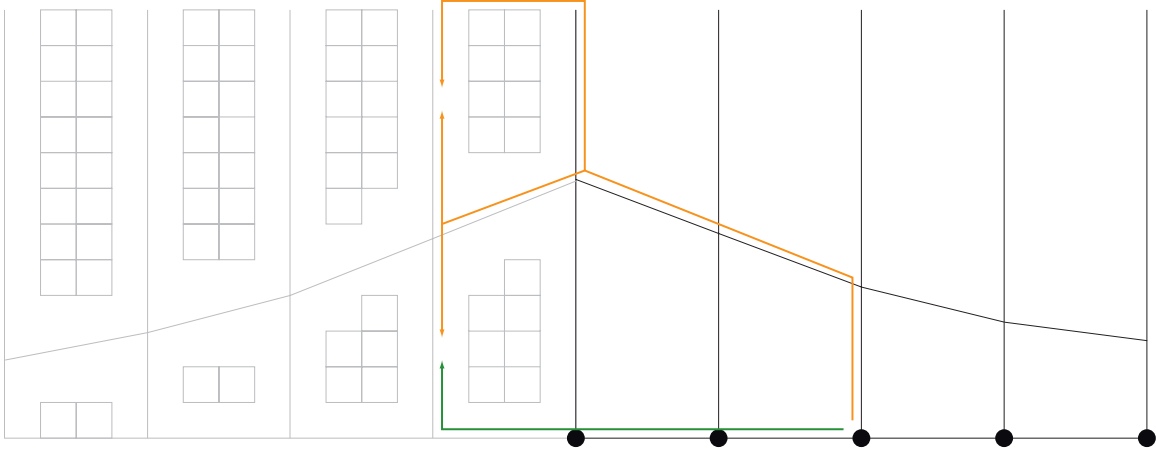


Figure 8: A rarely used and complicated travel pattern in the Inverted-V design. For some aisle combinations this is preferred to the travel patterns in Figure 4.

Locations at the top and bottom of a destination aisle are always visited in the same way, but for locations in the middle, we have a choice.

For a P&D point  $i$  in the right and picking aisle  $\ell$  in the left half-warehouse, we introduce  $L(i, \ell) = (i + \ell)a + b_\ell$  and  $U(i, \ell) = b_i + d_{i0} + d_{\ell 0}$ , which are the lower and upper shortest paths from the P&D point  $i$  to the intersection point  $b_\ell$ . If  $U(i, \ell) < L(i, \ell)$ , it is cheaper to access  $b_\ell$  from above, via the Inverted-V cross aisle. In this situation, it is better to use the cross aisle, which would produce the pattern given in Figure 8. This pattern creates two points of indifference in aisle  $\ell$ , one above and one below  $b_\ell$ . This holds for every aisle  $\ell$  such that  $U(i, \ell) < L(i, \ell)$  and  $1 \leq \ell < i$ . It is easy to see that for  $\ell \geq i$ , the lower path is best.

For the point of indifference  $q_{i\ell}^L$  below the cross aisle:

$$\begin{aligned}
 (\ell + i)a + q_{i\ell}^L &= b_i + d_{i0} + d_{\ell 0} + b_\ell - q_{i\ell}^L \\
 q_{i\ell}^L &= \frac{1}{2} \left[ b_i + d_{i0} + d_{\ell 0} + b_\ell - (\ell + i)a \right].
 \end{aligned}$$

and for the point of indifference  $q_{i\ell}^U$  above the cross aisle,

$$\begin{aligned} d_{\ell 0} + q_{i\ell}^U - b_\ell &= h - b_0 + \ell a + h - q_{i\ell}^U \\ q_{i\ell}^U &= \frac{1}{2} \left[ b_\ell + 2h - b_0 + \ell a - d_{\ell 0} \right]. \end{aligned}$$

As before, we must avoid imposing a bound for the slope of the Inverted-V cross aisle by defining,

$$\begin{aligned} \mathbf{q}_{i\ell}^L &= \max(w, \min(q_{i\ell}^L, b_\ell - w)) \\ \mathbf{q}_{i\ell}^U &= \max(b_\ell + w, \min(q_{i\ell}^U, h - w)). \end{aligned}$$

If  $L(i, x) < U(i, x)$ , the lower path is best and we have only one point of indifference above the cross aisle:

$$\begin{aligned} (i + \ell)a + q_{i\ell}^\ell &= b_i + d_{i0} + h - b_0 + \ell a + h - q_{i\ell}^\ell \\ q_{i\ell}^\ell &= \frac{1}{2} \left[ b_i + d_{i0} + 2h - b_0 - ia \right]. \end{aligned}$$

Again, we must avoid the potential for “negative distances,” so we define:

$$\mathbf{q}_{i\ell}^\ell = \max(b_\ell + w, \min(q_{i\ell}^\ell, h - w)).$$

For a fixed P&D point  $i$ , we choose the cheaper of the two travel patterns for every aisle  $\ell$  in the left half-space. Expected travel distance for the travel pattern with two points of

indifference (Figure 8) is,

$$\begin{aligned}
M_{i,\ell}(ab) &= \frac{1}{m(h-2w)} \left[ \int_0^{\mathbf{q}_{i\ell}^L} [(i+\ell)a+x]dx + \int_{\mathbf{q}_{i\ell}^L}^{b_\ell-w} [b_i+d_{i0}+d_{\ell 0}+b_\ell-x]dx \right. \\
&\quad + \int_{b_\ell+w}^{\mathbf{q}_{i\ell}^U} [b_i+d_{i0}+d_{\ell 0}+x-b_\ell]dx \\
&\quad \left. + \int_{\mathbf{q}_{i\ell}^U}^h [b_i+d_{i0}+h-b_0+\ell a+h-x]dx \right] \\
&= \frac{1}{m(h-2w)} \left[ \mathbf{q}_{i\ell}^L((i+\ell)a + \frac{\mathbf{q}_{i\ell}^L}{2}) + (b_\ell-w-\mathbf{q}_{i\ell}^L) \left[ b_i+d_{i0}+d_{\ell 0} \right. \right. \\
&\quad \left. \left. + \frac{1}{2}(b_\ell+w-\mathbf{q}_{i\ell}^L) \right] + (\mathbf{q}_{i\ell}^U-b_\ell-w) \left[ b_i+d_{i0}+d_{\ell 0} + \frac{1}{2}(\mathbf{q}_{i\ell}^U-b_\ell+w) \right] \right. \\
&\quad \left. + (h-\mathbf{q}_{i\ell}^U) \left[ b_i+d_{i0}+2h-b_0+\ell a - \frac{1}{2}(h+\mathbf{q}_{i\ell}^U) \right] \right].
\end{aligned}$$

For the travel pattern with a single point of indifference (Figure 4),

$$\begin{aligned}
M_{i,\ell}(bel) &= \frac{1}{m(h-2w)} \left[ \int_0^{b_\ell-w} [(i+\ell)a+x]dx + \int_{b_\ell+w}^{\mathbf{q}_{i\ell}^\ell} [(i+\ell)a+x]dx \right. \\
&\quad \left. + \int_{\mathbf{q}_{i\ell}^\ell}^h [b_i+d_{i0}+h-b_0+\ell a+h-x]dx \right] \\
&= \frac{1}{m(h-2w)} \left[ (i+\ell)a(b_\ell-w) + \frac{(b_\ell-w)^2}{2} \right. \\
&\quad \left. + (\mathbf{q}_{i\ell}^\ell-b_\ell-w) \left[ (i+\ell)a + \frac{1}{2}(\mathbf{q}_{i\ell}^\ell+b_\ell+w) \right] \right. \\
&\quad \left. + (h-\mathbf{q}_{i\ell}^\ell) \left[ b_i+d_{i0}+2h-b_0+\ell a - \frac{1}{2}(h+\mathbf{q}_{i\ell}^\ell) \right] \right].
\end{aligned}$$

The expected distance from P&D point  $i$  to a location in the leftmost region is

$$E[M_i] = \sum_{\ell=1}^m \min(M_{i,\ell}(bel), M_{i,\ell}(ab)).$$

The expected distance from the center P&D point is

$$E[C_0] = 2p_{R_0}E[R_0] + p_{S_0}E[S_0].$$

where,

$$\begin{aligned} E[R_0] &= \sum_{r=1}^m \frac{1}{m(h-2w)} \left[ \int_0^h (ra+x)dx - \int_{b_r-w}^{b_r+w} (ra+x)dx \right] \\ &= \sum_{r=1}^m \frac{1}{m(h-2w)} \left[ ar(h-2w) + \frac{h^2}{2} + 2b_r w \right] \\ E[S_0] &= \frac{1}{h-2w} \left[ \int_0^h xdx - \int_{b_0-w}^{b_0+w} xdx \right] \\ &= \frac{1}{h-2w} \left[ \frac{h^2}{2} - 2b_0 w \right]. \end{aligned}$$