# Staging Protocols for Unit-Load Crossdocking

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April 6, 2007

#### Abstract

To reduce inventory and transportation costs, many distributors use a logistics technique called *crossdocking*, in which products in a warehouse move directly from inbound to outbound vehicles, without storage in between. Although products are rarely stored outright, they are often staged on the dock temporarily to facilitate value-added services, to allow efficient loading of outbound trailers, or simply to accommodate unavoidable imbalances in flow. We identify, classify, and compare different protocols for staging pallets in a crossdock, and introduce a new kind of queue, called a *staging queue*, with which we model staging in the most common protocols.

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### 1 Introduction

Crossdocking is a logistics technique that effectively eliminates the inventory-holding function of a warehouse while still allowing it to serve its consolidation and order fulfillment functions. The idea is to transfer shipments directly from incoming to outgoing trailers without storage in between. Shipments typically spend less than 24 hours in a crossdock, sometimes less than an hour.

Crossdocks are essentially transshipment facilities to which trucks arrive with goods that must be sorted, consolidated with other products, and loaded onto outbound trucks. From a management perspective, crossdocking is a complex enterprise, involving extensive coördination between the distributor and its suppliers and customers (Schaffer, 1997). The crossdock must know which products are arriving in which trucks at which times for which customers, and, if there is a high degree of consolidation — as in the case of a PC distributor matching specific monitors with CPUs, for example — the crossdock must schedule trucks so as to avoid excessive congestion due to short term storage.

If a crossdock can have some short term storage, what differentiates it from a warehouse with a very high turnover rate? We answer, in a crossdock, the destination for an item is known before or determined upon receipt; in a warehouse, product is stored until a customer is identified. Because the customer is known upon arrival to a crossdock, there is no need to store products as inventory.

Crossdocking is attractive in the distribution industry for two reasons. For some distributors, crossdocking is a way to reduce inventory holding costs for sku's with stable, high demand. For others, crossdocking is a way to reduce inbound transportation costs. For example, individual outlets might receive shipments directly from vendors using less-than-truckload (LTL) or small package carriers, leading to excessive inbound transportation costs. Crossdocking is a way to consolidate those shipments to achieve truckload quantities.

## 1.1 Operations inside a crossdock

Crossdocks in the distribution and transportation industries take many forms. In the less-than-truckload (LTL) motor carrier industry, crossdocks are typically long, narrow facilities

with truck doors around almost the entire perimeter. Freight is moved by forklift and by workers pushing carts filled with freight and, with very few exceptions, there is no automated material handling.

Crossdocks in retail distribution are typically rectangular and wider than those in the LTL industry to accommodate product staging and value-added services. These facilities may use conveyors and sortation systems, but often most of the material handling is manual. Manual handling is common in retail distribution because it is easy to adjust the capacity of operations in response to seasonal fluctuations and market dynamics. Many retail distribution centers have small, low-volume crossdocking operations, typically in a corner of the warehouse, with trailer doors on each side, and material flow relatively isolated from the rest of the warehouse operations. Such are *not* the subject of this chapter: We are interested in high-flow facilities that are devoted entirely to crossdocking and that require temporary staging of shipments on the dock.

One way to divide crossdocking operations is by the handling units. In *case-pick cross-docking*, the distributor receives pallets of product and ships cases, or even individual items (*eaches* in industry parlance). For example, workers at one large retailer we visited receive pallets of product and place them directly into flow rack modules, from which other workers pick cases and send them to the shipping area via a conveyor system. The cases are loaded directly from shipping chutes into outbound trailers, and the product resides in the warehouse for only a few hours.

Unit-load crossdocking is strictly pallet in, pallet out, and so may also be called pallet crossdocking. Warehouse stores such as Costco and Sam's Club use this type of crossdocking because the retail outlets receive, and usually display, pallet quantities. At a typical unit-load crossdock, vendors call in advance to make appointments for deliveries, and the crossdock assigns a time window for the delivery. If the driver is late, he must make another appointment, typically the following day; if he is early, he waits outside the facility until his appointed time. As soon as the trailer is unloaded, a driver pulls away the empty trailer and another full trailer pulls up. This way, doors at the crossdock are almost always occupied with trailers being unloaded.

Material flow in a unit-load crossdock is relatively simple. Each shipment begins in its

arriving trailer and ends in its destination trailer. In the ideal case, workers take the pallets directly from inbound to outbound trailers, which reduces handling cost and keeps the dock clear for improved material flow. In practice, direct transfer rarely happens and pallets are staged because there is a need

- to perform value-added processes (labeling, pricing, etc.),
- to wait for other items of an order to arrive,
- to facilitate building tightly-packed loads in the outbound trailer, or
- to load in reverse order of delivery if there will be multiple stops.

Staging shipments on the dock creates problems. First, any staged shipment is handled multiple times, which adds to labor costs and increases risk of damage or loss. Second, it delays shipments in the facility, risking late delivery and consequent penalties. Delayed shipment also risks worker overtime when, as is common in many crossdocks supporting retail, all shipments must be processed everyday. Third, staged freight can create congestion and delay because there is less room for forklift drivers to maneuver. Fourth, staging requires space, which means a larger facility and associated costs.

## 1.2 Research questions

Patterns of material flow in a crossdock are the result of several design choices, such as where incoming and outgoing trailers are parked, the arrangement of temporary staging areas, how much information is known about shipments upon arrival, and the types of material handling equipment used (forklifts? conveyors? gravity flowrack?).

The effect of trailer placement on labor costs in a crossdock has been considered by several authors. Tsui and Chang (1990, 1992) consider layouts having all incoming trucks on one side and all outgoing trucks on the other. Crossdocks in the retail distribution industry are often arranged in this way, so that shipments flow directly from one side of the dock to the other, which facilitates orderly staging of shipments for value-added services. Bartholdi and Gue (2000) found that trailers generating the most worker activity — incoming trailers and the highest-flow destination trailers — should be located in the center of the dock on

both sides. In such a layout, freight moves in both directions across the dock and in both directions along the length of the dock, but is generally concentrated in the center region. Gue (1999) considered the effects of assigning incoming trailers to doors dynamically, based on their contents (commonly called the *spotting problem*). He found that when there is freight for relatively few (3–6) destinations per incoming trailer, there is significant benefit to adjusting the layout to account for intelligent spotting. Deshpande et al. (2007) simulated the operation of a crossdock in which incoming trailers are assigned to doors based on the destinations of shipments inside. Bartholdi and Gue (2004) investigated the best shape for crossdocks in the LTL industry. They found that small docks should be rectangular, larger docks should be T-shaped, and very large docks X-shaped.

Previous research has focused on the doors at which freight arrives and the doors at which it departs the crossdock, but has not looked at the details of movement—staging and sorting—as freight crosses the dock. This chapter studies the details of this movement, which we call the *staging protocol*. We have found different protocols in industry, which we list and categorize here. We analyze each as an engineering response to the particular information available about a shipment at the time of its arrival at the crossdock, and the particular material handling systems on the crossdock. We identify a number of design criteria and evaluate each protocol in light of them. Our goal is to understand the advantages and disadvantages conferred by each protocol, and to identify operational environments where each might be appropriate.

In the next section, we describe design criteria for a unit-load crossdocking operation, including both constraints and goals. In Section 3, we categorize staging protocols we have encountered in practice and describe how the subject firms executed those protocols and why. In Sections 4 and 5 we present throughput models for these protocols, including simulation and analytical models for two types of staging systems. We summarize our findings and give suggestions for design in Section 6.

## 2 Design criteria

How should a crossdock organize its material flow to reduce labor cost, support value-added services, and facilitate tightly-packed outbound loads? The answers to this question depend on what information is known about each shipment upon its arrival.

Information about arriving shipments affects material flows in a simple but important way. If freight is allocated to destinations and labeled before arriving at the crossdock, then it is possible, at least in principle, for workers to take freight directly from inbound to outbound trailers without intermediate staging. If the freight is not already labeled on arrival, then it must be staged on the dock, where other workers provide destination labels. These two types of crossdocking are commonly called *pre-* and *post-distribution*, respectively.

Post-distribution crossdocking incurs double-handling of freight but it enables the distributor to postpone allocation until shortly after the freight arrives at the crossdock, by which time the inventory situation of individual stores may have changed. As we will show, some staging protocols are inappropriate for pre- or post-distribution crossdocking. Whichever type of crossdocking (pre- or post-distribution), managers judge the suitability of a staging protocol based on several criteria, the most important of which is generally throughput, or, equivalently, labor cost.

In addition, the staging protocol must support whatever value-added processing is required of the crossdock. It may be that products must be processed by origin (so that products in an incoming trailer receive the same service) or by destination or by some other attribute, such as product type, color, or style, and so on. Staging protocols differ in which types of value-added processing they most naturally support.

Finally, staging protocols affect the efficiencies with which trailers are loaded. The percentage volume filled of a departing trailer is called the *load factor*; and it is important to achieve a high load factor, especially for the longer routes, because this reduces the number of trips in the long run. Some staging protocols make it easier to achieve high load factors.

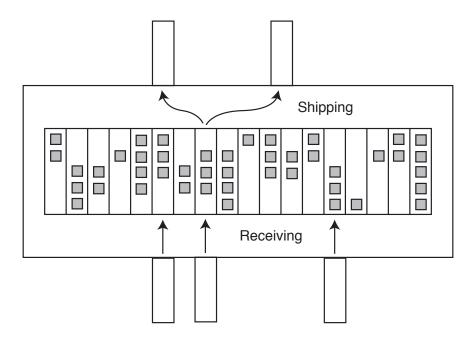


Figure 1: A crossdock operating a single-stage, sort-at-shipping protocol. Workers put pallets in lanes corresponding to the receiving doors. Workers in the shipping area sort pallets into appropriate shipping doors.

## 3 Staging protocols

In the *single-stage*, *sort-at-shipping* protocol workers pull pallets out of an arriving truck and put them in a lane outside the receiving door. Workers pull pallets out of the other end and deliver them directly to the appropriate outbound trailers. This method is appropriate for post-distribution crossdocking, when pallets must be labeled by destination upon arrival.

The advantage of a sort-at-shipping protocol is that the destination of a pallet need not be known when the worker unloads the freight from the trailer. This relieves the vendor of the burden of labeling pallets before shipping them and allows postponement of the allocation of freight to destinations. The crossdock may print labels anytime after the contents of an inbound trailer have been sent electronically; and workers apply them after pallets are in the staging area.

We have seen this protocol used by a large retailer of home improvement products. Orders from several vendors arrived "flow loaded" by product type and not by customer (e.g., model

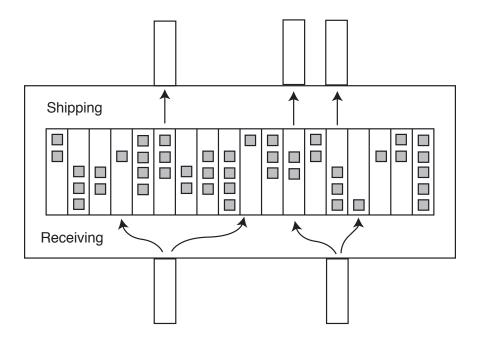


Figure 2: The sort-at-receiving protocol, which requires vendors to label incoming freight with its final destination.

A in the nose of the trailer, model B in the middle, model C in the tail). Products were unloaded and staged by the receiving door, where workers sorted products onto pallets by customer and other workers delivered the pallets to shipping doors, effectively sorting at shipping.

In a *single-stage*, *sort-at-receiving* operation, workers take pallets directly from receiving door to the lane associated with the proper shipping door. Note that this works only if bar codes or other labels have been attached by the vendor. The advantage of staging by shipping door is that workers in shipping have a better view of what freight is available for loading, and so can achieve a tighter pack of freight while loading, thus reducing transportation costs in the long run.

We have seen this protocol at Maritime-Ontario Freight Lines, an LTL carrier in Canada, where the enormous distances make linehaul the largest component of operating cost. Accordingly, it is important to reduce the number of trailers by building tight loads. Pallets at Maritime-Ontario are staged in lanes corresponding to destination so that workers in the shipping area can "cherry-pick" pallets to tightly pack the outgoing trailers.

Figure 3 shows the single-stage, double-sort protocol, which combines the previous pro-

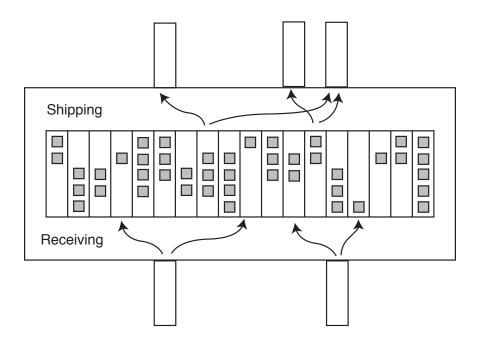


Figure 3: The single-stage, double-sort protocol.

tocols to allow processing according to a criterion independent of the origin or destination trailer. We have not seen this protocol in practice, but it might be appropriate for predistribution operations where load factor is not a major concern and there is a need for processing according to product type.

Figure 4 is a two-stage protocol. Multi-stage protocols make it easier to pack trailers tightly, because workers in shipping can pick from among several pallets in shipping queues, while still allowing value-added processing by other criteria. The disadvantage is that pallets are handled multiple times. Furthermore, the crossdock must be wider to accommodate the additional queues, which increases both fixed costs and labor cost due to travel. (In Section 4 we consider the throughput implications of having multiple staging queues.)

This protocol was used at the Costco distribution center near Tracy, California. At the time of our last visit, the facility had negotiated pre-distribution labeling agreements only with its largest vendors. Nevertheless, they had full pre-distribution operations as a goal in hopes of reducing labor costs and cycle time for shipments on the dock. At this facility, it was especially important to reduce cycle times because outbound trailers had specific times at which they had to leave in order to make their destinations on time.

We have seen other protocols as well. Free staging is used almost universally in the

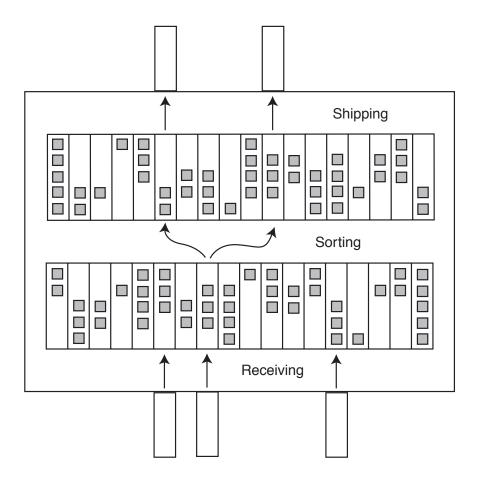


Figure 4: A two-stage crossdock. Workers put pallets in lanes corresponding to the receiving doors; a second team of workers sorts pallets into shipping lanes, from which a final team loads them onto outbound trailers.

Table 1: Comparison of staging protocols:  $\bullet$  indicates an advantage;  $\bullet$  a slight benefit;  $\bigcirc$  a disadvantage.

Criterion	SaS	SaR	DS	MS
Information requirements	•	$\circ$	$\circ$	•
Loading efficiency	0	•	$\circ$	•
Handling cost	•	•	•	$\bigcirc$
Value-added services	•	$lackbox{0}$	•	•

less-than-truckload (LTL) industry. Workers in an LTL crossdock place pallets in the center of the dock opposite or nearly opposite the appropriate shipping door. Entry to and from the staging area is from the side toward the shipping door. LTL carriers use this method rather than a single-stage protocol because they usually handle pallets and loose pieces, and they always have to handle oddly shaped freight which is not amenable to a strictly defined staging lane. This method also has the advantage that loading directly into the shipping door, when possible, is very easy because there is a clear path to the door from the receiving area.

In a *stage-by-door* operation workers put pallets in gaps between shipping doors. This protocol is possible only when the distance between door centers is wide enough to allow a pallet between doors, and this is one of the tradeoffs of using this method. This method combines the advantage of free staging—that direct loading is easy—with the advantage of sort-at-receiving—that loaders can pick loads for higher load averages. We have seen this method used at a crossdock operated by a third-party logisitics provider for Sam's Club.

Table 1 provides a brief visual comparison of four protocols appropriate for retail distribution. Sort-at-shipping (SaS) requires little in terms of information transfer with vendors because pallets are staged outside receiving doors, where they can be labeled after unloading. This is an advantage  $(\bullet)$ . However, this protocol makes efficient loading difficult  $(\bigcirc)$  because workers must load shipments upon delivery to the shipping door—there is no opportunity to carefully select loads for a tight pack. SaS has acceptable handling cost  $(\bullet)$  because pallets are touched only twice (receiving door to staging; staging to shipping door). Value-added

services are possible  $(\bullet)$ , but only by origin; that is, distinguishing the service by destination or another criterion is difficult.

Sort-at-receiving (SaR) requires the crossdock to receive pallets with labels attached, so it has a burdensome requirement for IT coordination with vendors ( $\bigcirc$ ). Because pallets are staged according to their destinations, loaders are able to carefully select pallets for tight packing ( $\bullet$ ). Handling cost is similar to sort-at-shipping ( $\bullet$ ), and value-added services are possible only by destination ( $\bullet$ ).

Double-sort (DS) crossdocking seems to combine the *dis*-advantages of the previous two protocols, and even has a slightly higher handling cost ( $\mathbb{O}$ ) due to the additional travel associated with double-sort. Double-sort has the advantage of allowing a distributor to perform value-added services on a basis other than origin or destination ( $\mathbb{O}$ ).

Multiple-stage (MS) requires little information transfer with vendors ( $\bullet$ ) and provides the opportunity for high load factors on outbound trucks ( $\bullet$ ), but this comes at the cost of additional handling ( $\bigcirc$ ). Value-added services in a multiple-stage operation are possible both with respect to origin and destination ( $\bullet$ ).

## 4 Models

As our figures suggest, most unit-load crossdocks have staging areas where pallets are placed on the floor for temporary storage. In the protocols we describe, pallets are sorted into lanes, and these lanes act as finite buffers in which the pallets queue. If the last position in a lane is occupied, the lane is blocked, and this has an effect on throughput.

In an ordinary finite queue, a customer may join as long as the number of customers in the queue is less than the number of positions in the buffer. This is because customers move forward after each service, leaving room for new customers in the rear. We call this a move-to-front queue. Queues in a unit-load crossdock with floor staging operate differently because pallets in the queue do not move forward after each service. We call this type of queue a staging queue.

Figure 5 shows how a staging queue works. Workers deposit pallets in the forward-most empty position in the queue. As the server pulls a pallet from the queue, the remaining



Figure 5: How a staging queue works. Top: Pallets occupy positions 3–5. Bottom: After 2 arrivals and 1 service, positions 4–7 are occupied.

pallets do not move and so no room is made for additional pallets to join the queue. (The return aisles of rental car lots are another example of staging queues, in which returning customers park in the forward-most empty space in one of the lanes. As each car is served, an attendant drives away the forward-most car in a lane, but cars in the rear do not advance because they are unoccupied.)

#### 4.1 A single-stage model

We consider first a single staging queue with a single server, which corresponds to the sortat-shipping, sort-at-receiving, and double-sort protocols. We assume that workers deposit pallets in the forward-most empty position, and the server removes the forward-most pallet from the other side, as in Figure 5. Note that in our model,

#### **Lemma 1** Pallets in a staging queue must be contiguous.

This is because pallets enter the queue from the rear and occupy the forward-most position, and only the forward-most pallet from the front may be served. Moreover, the block of pallets forms a backward propagating "wave" that either "breaks early" (meaning that it never reaches the last position and blocks the queue) or "beaches" (it eventually blocks the queue until cleared).

To make our analytical model tractable, we assume that arrivals balk if they find the queue full. In Section 5 we use simulation to consider the case of blocking instead of balking.

We model the staging queue as a continuous time Markov chain. Figure 6 illustrates the state space for a 3-position staging queue. Formally, we say the system is in state (i, j) when the rearward-most occupied position is i and the forward-most occupied position is j (therefore  $i \geq j$ .) If no positions are occupied and the server is busy, the system is in state (0,0); otherwise, the server is idle and the system is empty and in state e.

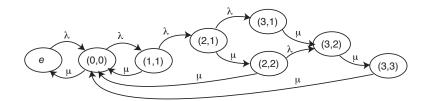


Figure 6: The state space diagram for a 3-pallet staging queue.

Let  $\pi_{ij}$  be the steady state probability that the system is in state (i, j). For the 3-pallet case we see that,

$$\lambda \pi_{e} = \mu \pi_{00},$$

$$(\lambda + \mu)\pi_{00} = \lambda \pi_{e} + \mu(\pi_{11} + \pi_{22} + \pi_{33}),$$

$$(\lambda + \mu)\pi_{11} = \lambda \pi_{00},$$

$$(\lambda + \mu)\pi_{21} = \lambda \pi_{11},$$

$$(\lambda + \mu)\pi_{22} = \mu \pi_{21},$$

$$\mu \pi_{31} = \lambda \pi_{21},$$

$$\mu \pi_{32} = \lambda \pi_{22} + \mu \pi_{31},$$

$$\mu \pi_{33} = \mu \pi_{32}, \text{ and}$$

$$\pi_{e} + \sum_{i,j} \pi_{ij} = 1.$$

The transition probabilities are

$$\pi_e = \frac{\mu}{\lambda} \pi_{00}, \tag{1}$$

$$\pi_{11} = \frac{\lambda}{\lambda + \mu} \pi_{00}, \tag{2}$$

$$\pi_{21} = \frac{\lambda^2}{(\lambda + \mu)^2} \pi_{00}, \tag{3}$$

$$\pi_{22} = \frac{\lambda^2 \mu}{(\lambda + \mu)^3} \pi_{00}, \tag{4}$$

$$\pi_{31} = \frac{\lambda^3}{\mu(\lambda + \mu)^2} \pi_{00}, \tag{5}$$

$$\pi_{32} = \frac{2\lambda^3 \mu + \lambda^4}{\mu(\lambda + \mu)^3} \pi_{00}, \tag{6}$$

$$\pi_{33} = \frac{2\lambda^3 \mu + \lambda^4}{\mu(\lambda + \mu)^3} \pi_{00}, \tag{7}$$

where

$$\pi_{00} = \left(1 + \frac{\mu}{\lambda} + \frac{\lambda}{\lambda + \mu} + \frac{\lambda^2}{(\lambda + \mu)^2} + \frac{\lambda^2 \mu}{(\lambda + \mu)^3} + \frac{\lambda^3}{\mu(\lambda + \mu)^2} + \frac{4\lambda^3 \mu + 2\lambda^4}{\mu(\lambda + \mu)^3}\right)^{-1}.$$

Arrivals are served anytime the last position in the queue is not occupied, so the effective system throughput is  $\lambda_{\text{eff}} = (1 - \pi_{31} - \pi_{32} - \pi_{33})\lambda$ .

Notice that if we add a position to the example staging queue (giving it 4 positions), Equations 1–4 are the same; we need only derive new equations for what were previously blocking states  $(\pi_{3j})$  and the new blocking states  $(\pi_{4j})$ , and then recompute  $\pi_{00}$  to obtain the probabilities. Following are the general results.

**Theorem 1** In a staging queue with n = 1 position, steady state probabilities are

$$\pi_e = \frac{\mu^2}{\mu^2 + \lambda \mu + \lambda^2},$$

$$\pi_{00} = \frac{\lambda \mu}{\mu^2 + \lambda \mu + \lambda^2},$$

$$\pi_{11} = \frac{\lambda^2}{\mu^2 + \lambda \mu + \lambda^2}.$$

*Proof.* Proof. Follows directly from the state diagram and some arithmetic.

**Theorem 2** In a staging queue with  $n \ge 2$  positions, steady state probabilities  $\pi_{ij}$  are, for non-blocking states,  $\pi_e = (\mu/\lambda)\pi_{00}$  and  $\pi_{ij} = r_{ij} a_{ij} \pi_{00}$ , where

$$r_{ij} = r_{i-1,j} + r_{i,j-1}, \text{ (where } r_{ij} = 0 \text{ for } i < j, r_{i0} = 0, \text{ and } r_{i1} = 1),$$

$$a_{ij} = \left(\frac{\lambda}{\lambda + \mu}\right)^{i} \left(\frac{\mu}{\lambda + \mu}\right)^{j-1} \text{ (for } i = 1 \dots n - 1, j = 1 \dots i),$$

$$\pi_{00} = \left(1 + \mu/\lambda + \sum_{i=1}^{n-1} \sum_{j=1}^{i} r_{ij} a_{ij} + \lambda/\mu \sum_{k=1}^{n-1} \sum_{j=1}^{k} r_{n-1,j} a_{n-1,j} + \lambda/\mu \sum_{j=1}^{n-1} r_{n-1,j} a_{n-1,j}\right)^{-1}$$

and for blocking states,

$$\pi_{n1} = (\lambda/\mu)\pi_{n-1,1},$$

$$\pi_{ni} = (\lambda/\mu)\pi_{n-1,i} + \pi_{n,i-1}, \text{ (for } i = 2...n-1), \text{ and}$$

$$\pi_{nn} = \pi_{n,n-1}.$$

*Proof.* For  $\pi_e$  we appeal to the example state diagram in Figure 6 directly. The expression for  $\pi_{00}$  comes from  $\pi_e + \sum_{ij} \pi_{ij} = 1$  and the recursive expressions for the blocking states. For the remaining non-blocking states, we prove the result by induction. Consider the state (1,1) in the state diagram,

$$\pi_{11} = \frac{\lambda}{\lambda + \mu} \pi_{00}$$
$$= (r_{10} + r_{01}) \frac{\lambda}{\lambda + \mu} \pi_{00},$$

which is the result.

Now assume the result is true for  $\pi_{i,j-1}$  and  $\pi_{i-1,j}$ . Notice from the state diagram that for all non-blocking states except state e the relationship  $(\lambda + \mu)\pi_{ij} = \mu\pi_{i,j-1} + \lambda\pi_{i-1,j}$  holds, where  $\{\pi_{ij} = 0 : i < j \text{ or } j = 0\}$ . Then,

$$(\lambda + \mu)\pi_{ij} = \mu \pi_{i,j-1} + \lambda \pi_{i-1,j}$$

$$\pi_{ij} = \frac{\mu}{\lambda + \mu} \pi_{i,j-1} + \frac{\lambda}{\lambda + \mu} \pi_{i-1,j}$$

$$= \frac{\mu}{\lambda + \mu} \left( \frac{r_{i,j-1} \lambda^{i} \mu^{j-2}}{(\lambda + \mu)^{i+j-2}} \right) \pi_{00} + \frac{\lambda}{\lambda + \mu} \left( \frac{r_{i-1,j} \lambda^{i-1} \mu^{j-1}}{(\lambda + \mu)^{i+j-2}} \right) \pi_{00}$$

$$= (r_{i-1,j} + r_{i,j-1}) \frac{\lambda^{i}}{(\lambda + \mu)^{i}} \frac{\mu^{j-1}}{(\lambda + \mu)^{j-1}} \pi_{00}.$$

For blocking states, we appeal directly to the example state diagram.

# Corollary 1 The effective throughput for an n-position staging queue is $\lambda_{eff} = (1 - \sum_{j} \pi_{nj})\lambda$ .

The model has several limitations with respect to crossdocking operations in practice. For example, crossdocks usually have two staging lanes per trailer, giving workers in receiving two queues into which they can drop a pallet. Also, workers arriving to a blocked queue take action to clear the block, such as notifying workers in the shipping area or clearing the block themselves.

We also ignore the effect on arrival and service rates of changing travel distance due to pallets moving in the queue. For our purposes, there are two types of travel in the crossdock: travel to and from the queue, and travel within the queue. Because crossdocks are typically much longer than they are wide, travel within the queue, which is at most the length of

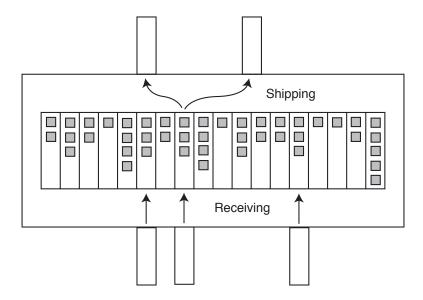


Figure 7: When pallets are staged in a move-to-front device such as flow rack, they roll forward to the frontmost open positions.

10–15 pallets in practice, is much less than travel to and from the queue, which can be as much as several hundred feet. Thus the effect of changes in travel within the queue has a negligible effect on the arrival and service rates to the queue.

We assume interarrival and service times are exponentially distributed mostly as a matter of analytical tractability; however, we also note that there are many sources of variance for worker rates, and this argues at least for a distribution with high variance. Examples of worker variance include varying travel distances, time to label or inspect pallets, difficulty loading or unloading pallets, and downtime due to breaks or equipment malfunctions.

## 4.2 Move-to-front queues

Some crossdocks use flow rack or inclined rollers for pallet storage. In these devices, pallets automatically roll forward after every service, just as milk in a grocery store rolls forward when a carton is removed (see Figure 7). How much better is a move-to-front queue than a staging queue?

We can describe the state of a move-to-front queue with a single variable describing which is the rearward-most occupied position. Let a queue containing i pallets be in state i. It is easy to show that for a move-to-front queue with n pallet positions and  $\rho = \lambda/\mu$ ,

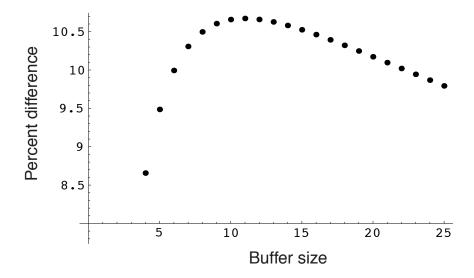


Figure 8: The move-to-front queue has higher throughput than staging queue for all buffer sizes, with the greatest difference at size 11.

 $\pi_e = \rho^{-1}\pi_0$ ,  $\pi_i = \rho^i\pi_0$  and  $\pi_0 = (\rho^{-1} + 1 + \rho + \rho^2 + \ldots + \rho^n)^{-1}$ . As before, the system is producing whenever the queue is not full, so effective system throughput is  $\lambda_{\text{eff}} = (1 - \pi_n)\lambda$ . Observe that because both queues block with a single pallet,

**Lemma 2** A move-to-front queue with one position is equivalent to a staging queue with one position.

Also,

**Lemma 3** For both the move-to-front and staging queues,  $\lambda_{eff} \rightarrow \lambda$  as  $n \rightarrow \infty$ .

This is true because as the buffer size goes to infinity, neither system blocks at all, and throughput is as high as possible.

Lemmata 2 and 3 suggest that for very small and very large queues, the staging and move-to-front queues behave similarly. Figure 8 shows the percent difference in  $\lambda_{\text{eff}}$  between a move-to-front queue and a staging queue for buffer sizes in between, when  $\lambda = \mu$ .

**Remark 1** A move-to-front queue has greater throughput than a staging queue for all buffer sizes greater than one and the maximum percent difference occurs at buffer size 11.

It is interesting to note that a 48-foot trailer is 12 pallets long, and consequently, staging areas are often about that length as well. The remark suggests that using a move-to-front storage device can increase system capacity by as much as 11%, but such devices also have several disadvantages. First, because it occupies floor space, a storage device obstructs material flow patterns that could otherwise be used to "cut corners" and make workers more efficient. Second, storage devices are fairly inflexible, potentially making changes to the dock difficult. Third, storage devices prevent workers on the shipping side from selecting pallets not at the head of the queue for loading, potentially reducing load average per trailer and increasing transportation costs. Fourth, many storage devices have a high initial cost, while floor staging has none.

## 5 Two-stage model

A two-stage system achieves the advantages of both staging by receiving and staging by shipping, but at what cost? To gain insight, we simulated a tandem staging queue system in which departures from the first queue become arrivals to the second. We ran two scenarios: in the first, arrivals to the shipping queue balk if it is full; in the second, the server for the receiving queue is blocked until the shipping queue is cleared. In each scenario, we set  $\lambda = \mu_1 = \mu_2 = 0.5$ , where  $\mu_1$  and  $\mu_2$  are the mean service rates for queues 1 and 2 respectively.

Figure 9 compares a single queue with n positions with tandem systems (each queue having n/2 positions) for the blocking and balking cases.

**Remark 2** A two-stage staging system has significantly lower throughput than a single-stage system when entities block between stages.

The implication for crossdock design is that, while a two-stage system offers the dual advantages of staging by receiving and by shipping, these advantages come at a cost of lower throughput. In practice this would be realized with higher levels of congestion as throughput increases, or with higher labor costs.

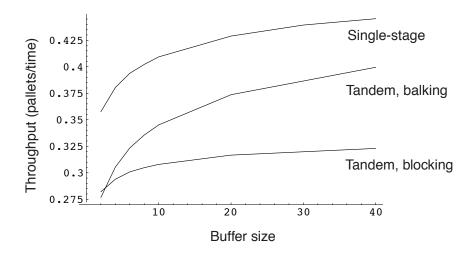


Figure 9: A single-stage system has significantly higher throughput than either tandem system with the same number of positions.

## 6 Conclusions

There are several ways to organize staging within a unit-load crossdock. Each protocol we presented in this chapter has a distinctive set of advantages and disadvantages, which determines its suitability for the information and logistics environment faced by a distributor.

The sort-at-shipping protocol is appropriate for distributors that do not have sophisticated information coordination with their suppliers, or for those with many suppliers, where such coordination is not practical. This protocol has the advantage of single-stage queueing, and so handling costs are relatively low. Sort-at-receiving requires the distributor to receive labeled unit-loads, and therefore requires a high degree of information coordination. It too has low handling costs due to single stage queueing. The dual-sort protocol provides the distributor the ability to perform value-added services according to a criterion other than the origin or destination of the shipment. The multiple-stage protocol combines advantages of sort-at-shipping and sort-at-receiving, but at the cost of additional handling and facility costs.

We found that although staging queues block more often than move-to-front queues, such as those formed by flow rack, the difference in throughput is less than about 11 percent in the worst case. We believe this argues strongly against using move-to-front storage devices for unit-load crossdocking, especially considering the many other disadvantages, such as high

initial cost, lack of flexibility, and obstruction of material flows.

Our results also suggest that multiple-stage systems, while having important operational advantages, do suffer significantly lower throughput than an equivalent single-stage system. This makes an important point about the value of information in a crossdocking logistics system. If a firm invests in the information systems and vendor relationships required for pre-distribution operations, it can take advantage of single-stage crossdocking, including higher throughput and lower labor requirements. The operations manager at one two-stage crossdock we visited stated that his firm would prefer to operate a single-stage system, were they able to establish the necessary information links with all of their vendors.

## Acknowledgements

We thank the managers and engineers of WalMart, Sam's Club, Costco, The Home Depot, Maritime-Ontario, and Yellow Transport for allowing us to study their crossdocking operations. We also appreciate the support of the Office of Naval Research (Bartholdi, Gue), the National Science Foundation (Bartholdi, Gue), and The Supply Chain & Logistics Institute at Georgia Tech (Bartholdi).

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