Turnover-Based Storage in Non-Traditional Unit-Load Warehouse Designs

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Abstract

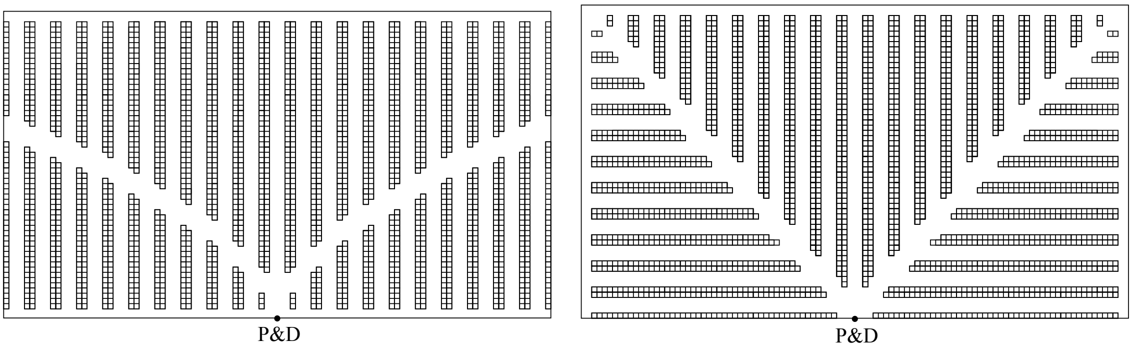
We investigate the effect of assigning the most-active items to the best locations in unit-load warehouses with non-traditional aisles. Specifically, we report the performance of flying-V and fishbone designs when products exhibit different velocity profiles. We consider both single- and dual-command operations in a warehouse where receiving and shipping are located at the midpoint of one side of the warehouse. For dual-command operations, a fishbone design shows similar reductions in travel distances for both random and turnover-based storage policies. The fishbone designs that provide the best performance have a diagonal cross aisle that extends to the upper corners of the picking space, and are approximately half as tall as they are wide. In general, warehouse design parameters that perform best under random storage also perform well under turnover-based storage.

1 Introduction

Warehouses are designed to supply customer needs, and can also provide a buffer against variable demand. In most cases, customer demand is skewed, meaning some products have higher average demand than others. To increase the efficiency of storage and retrieval operations when the demand is skewed, a common approach is to locate the shipping and receiving docks on the same side of the warehouse (Bartholdi and Hackman, 2007) and store the fast-moving items close to the dock doors. Turnover-based storage policies allocate items to dedicated storage locations based on their turnover frequency (items stored or retrieved per time period), where the items with the highest turnover are stored in the most convenient storage locations.

Another way to improve efficiency is to arrange the picking aisles so that travel in the warehouse is efficient. Gue and Meller (2009) introduced the flying-V and fishbone designs (Figure 1) to reduce single-command travel under a random storage policy. In the flying-V, picking aisles are parallel and there are orthogonal cross aisles at the top and bottom of the warehouse. The position and orientation of the middle cross aisle is determined by minimizing the expected travel distance from a single pickup and deposit (P&D) point located at the center of the bottom side to a single random location in the warehouse. For reasonable values of cross aisle width, the optimal shape of the cross aisle is V-shaped, with the vertex at the P&D point Gue and Meller (2009). The aisle appears to be curved, however each segment of the cross aisle between picking aisles is piecewise linear. Improvement in single-command travel under a random storage policy is approximately 10% when compared to an equivalently-sized (same number of pallet locations) traditional warehouse with no middle cross aisle.

The fishbone design has orthogonal cross aisles at the top of the warehouse and on the left and right edges. The middle cross aisle is diagonal and straight, with vertical picking aisles above and horizontal picking aisles below. The slope of the middle cross aisle is determined by minimizing the expected distance from the P&D point to a single random location in the warehouse. Under a random storage policy, the fishbone design reduces single-command travel by up to 20%, and dual-command travel by 10–15% (Pohl et al., 2009c) when compared to equivalently-sized traditional warehouses.



(a) Flying-V cross aisle (b) Fishbone aisles

Figure 1: Non-traditional aisle layouts

To date, the authors know of four implementations of non-traditional aisles in practice, each a variation of the fishbone design (Meller and Gue, 2009, describe two of the four). In at least one of these warehouses, products are assigned to locations roughly according to their demand rather than randomly. Managers at this facility believed that the concepts behind the fishbone design would confer acceptable benefits even though the random storage assumption was violated. The objective of this paper is to determine the effectiveness of the flying-V and fishbone designs under a turnover-based storage policy for both single-command and dual-command operations. We assume a single P&D point in the center of one side. For the impact of the single P&D point assumption on the flying-V design, the interested reader is directed to Ivanović et al. (2010), where the authors illustrate that the improvement of a flying-V layout decreases as the number of P&D points increase under random storage.

In turnover-based storage, fast-moving items are stored in the most convenient or desirable locations, where the desirability of the location depends on the warehouse design and operational policies. For single-command travel, the convenience of a location is defined by its distance from the P&D point. In this paper we refer to this as the *distance-based slotting strategy*, which minimizes the expected single-command travel distance (Francis et al., 1992).

Figure 2 illustrates the optimal slotting strategies for single-command travel in three traditional aisle layouts, which we identify as Layouts A, B and C. Layout A has parallel picking aisles and orthogonal cross aisles at each end of the picking aisles, while Layout B has a third cross aisle inserted halfway along the picking aisles. Layout C is similar to Layout B, however the aisle structure is rotated 90 degrees. The intensity of shading in the diagrams indicates the distance along the aisles from the P&D point to a particular pallet location, and therefore also indicates where items should be stored based on their turnover frequency. Note that the shape of the contours for these equivalently-sized warehouses is triangular, due to rectilinear travel.

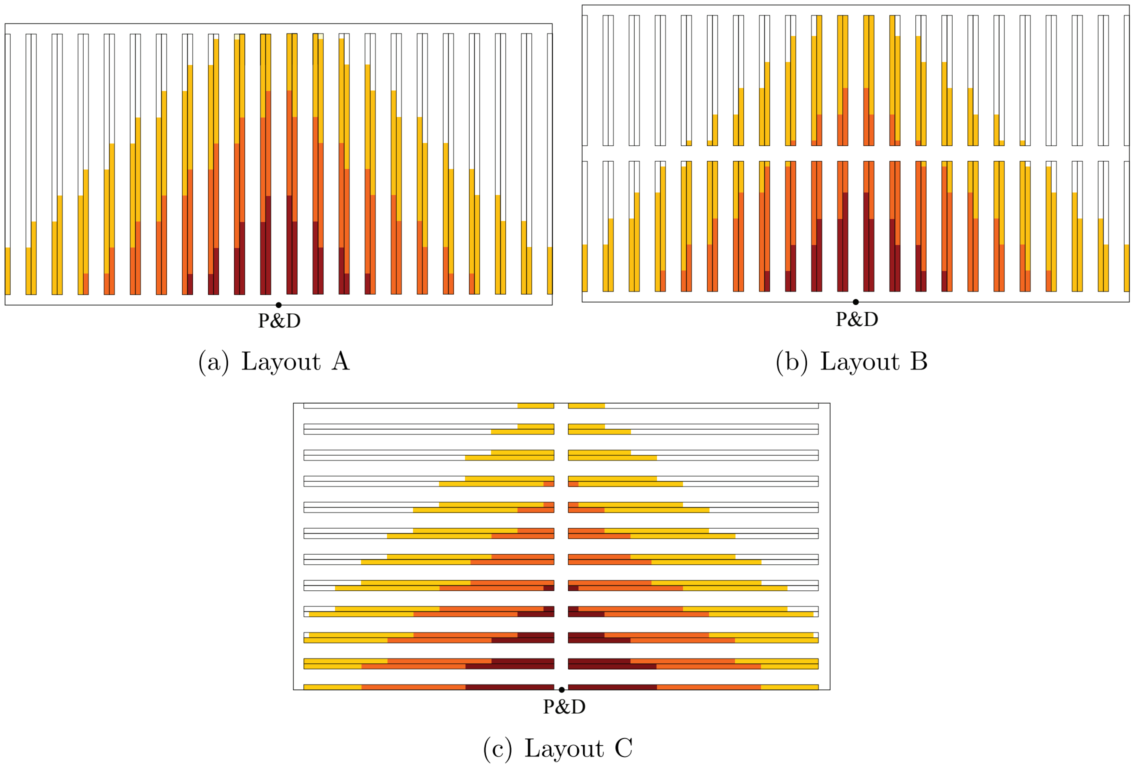
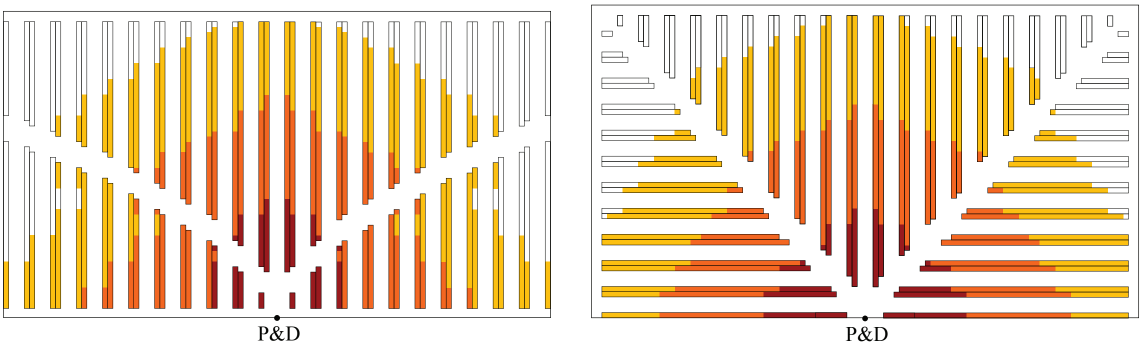


Figure 2: Distance-based slotting strategies for traditional layouts — optimal for single-command travel

When we apply the distance-based slotting strategy to the flying-V and fishbone layouts, the contours are not triangular, because the travel is no longer strictly rectilinear. The optimal slotting strategy for single-command travel in a flying-V warehouse has contours with a "mushroom" shape, as indicated in Figure 3(a). The contour shape for the fishbone design is approximately a semicircle (Figure 3(b)); because travel paths in a fishbone warehouse are close to Euclidean.



(a) Flying-V cross aisle (b) Fishbone aisles

Figure 3: Distance-based slotting strategies for flying-V and fishbone layouts — optimal for single-command travel

Figure 4 shows the percentage of pallet locations versus distance from the P&D point for equivalently-sized fishbone, flying-V and Layout A warehouses. We see that the fishbone design dominates both flying-V and Layout A; i.e., for any given distance from the P&D point, the fishbone design has an equal or greater number of pallet positions within that distance than either of the other two designs. The flying-V design is better than Layout A for distances greater than 30 pallets. It appears that for this example, the greatest benefit in travel is achieved for the pallet positions that are far from the P&D point. Distances in the flying-V and fishbone designs are reduced by Euclidean travel along the cross aisles, however the added space consumed by the cross aisles must also be traversed. We propose that for reasonable cross aisle widths, the benefit outweighs the penalty of crossing the cross aisle, particularly for relatively large warehouses.

Figure 4 indicates that the flying-V and fishbone designs are potentially good choices for a turnover-based storage system with single-command operations; however, we also consider dual-command operations. Although the distance-based slotting strategy is optimal for single-command travel, the best strategy for dual-command operations is not so evident. In a dual-command cycle, travel consists of both single-command travel and the travel between locations. The distance-based slotting strategy optimizes the former, but ignores the latter. Pohl et al. (2009b) find, using heuristic search methods, that the resulting contours for travel-between are often very different from the contours shown in Figures 2 and 3. The

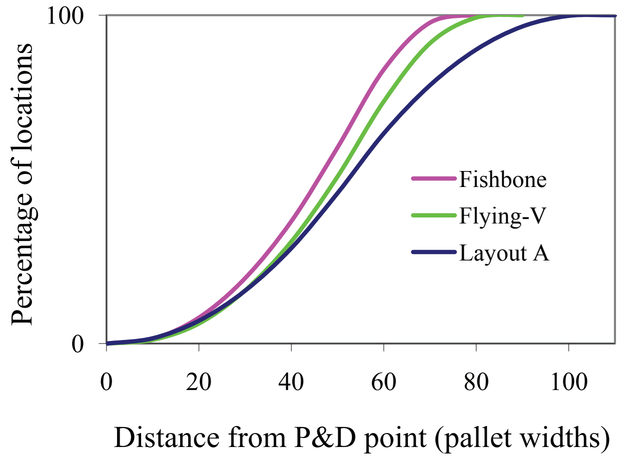


Figure 4: Percentage of pallet positions vs distance from P&D

distance-based strategy is therefore likely not optimal for dual-command travel, because it is a composite of single-command travel and travel-between. However, the best slotting strategies found for dual-command travel (again using heuristic search methods) resulted in improvements in expected travel distances of 1% or smaller over the distance-based strategy (Pohl et al., 2009b). Because these differences are small, we consider the distance-based slotting strategy a good, albeit not optimal, strategy for dual-command travel.

In the next section we review the relevant literature. In Section 3 we present the mathematical models for single- and dual-command travel distance, and detail our assumptions for warehouse design in Section 4. We then compare performance of the flying-V and fishbone designs to the three traditional warehouse designs, over a range of warehouse sizes, for both single-command operations (Section 5) and dual-command operations (Section 6). We offer conclusions in Section 7.

2 Literature Review

Francis et al. (1992) describe three general types of storage allocation policies: (1) random storage, which makes the most efficient use of space; (2) turnover-based storage, which reduces travel by dedicating the most convenient storage locations to items with the highest turnover frequency, but requires more storage space than random storage because space must be reserved for the maximum inventory of each item; and (3) class-based storage, which is a hybrid of random and turnover-based storage, and has some of the benefits of each. In class-based storage, products are divided into classes based on their turnover rates and each class is allocated a particular region of the warehouse. Within each region, a random storage policy is used. Product allocation has been studied by many researchers (see de Koster et al. (2007) and Gu et al. (2007)). The efficiency of workers is enhanced if a computerized warehouse management system is used to implement these storage allocation policies, as well as facilitate dual-command operations.

Goetschalckx and Ratliff (1990) consider shared storage and show that a duration-of-stay-based policy is optimal under an assumption of perfectly balanced inputs and outputs. The duration-of-stay approach requires arrival/departure information on individual items of a particular product, whereas the turnover-based and class-based storage policies require only turnover rate information at the product level. In this paper we assume we know only product information.

One version of a turnover-based storage policy is the cube-per-order index (COI) rule (Heskett, 1963). The COI is the ratio of an item's maximum allocated storage space to the number of storage/retrieval operations per unit time. Items are sorted by their COI and those with the smallest ratios are allocated the most convenient storage locations. In unit-load operations, a turnover-based policy is essentially the same as the COI policy if two conditions are met: all units are the same size, and if multiple pallets of the same item are stored, they are treated as separate items with the demand appropriately apportioned between items (Gu et al., 2007).

For single-command travel, the convenience of a location in a turnover-based storage system is determined by its shortest-path distance from the P&D point (Mallette and Francis, 1972; Harmatuck, 1976; Francis et al., 1992). When more than one location is visited in a single trip, such as in dual-command and order picking operations, the best slotting strategy is not known.

Malmborg and Krishnakumar (1987, 1990) study the COI rule for dual-command operations, concluding that the COI rule (and distance-based slotting) is optimal via a proof based on exchanging the location of two items (and arguing with an analytical expression that such an exchange cannot improve the solution). Pohl et al. (2009b) provide counter-examples to their proof by showing that exchanging two items may actually result in a net decrease in expected dual-command travel distance.

Turnover-based storage has been investigated in the order-picking environment by several authors. Jarvis and McDowell (1991) show that a within-aisle slotting scheme is optimal for the traversal routing policy in Layout A; Petersen and Schmenner (1999) and Hwang et al. (2004) consider several routing and storage policies in Layout A; and Caron et al. (2000a,b) consider Layout C.

Most of the research on class-based storage addresses automated storage/retrieval systems (AS/RSs), and is typically concerned with determining the number of classes and the boundaries of the warehouse regions. Graves et al. (1977) and Kouvelis and Papanicolaou (1995) derive analytical solutions for class boundaries with 2 or 3 classes; and Rosenblatt and Eynan (1989) and Eynan and Rosenblatt (1994) address the n-class case. Refer to recent survey papers (de Koster et al., 2007; Gu et al., 2007) for more examples.

Existing research on storage allocation policies addresses either AS/RSs or one of the traditional aisle layouts of Figure 2. This paper considers turnover-based storage policies in the flying-V and fishbone layouts, and compares the travel distances to those in the traditional layouts (Layouts A, B and C).

3 Warehouse Models

We define a warehouse as a set of picking aisles, each having a discrete number of locations. We consider only the shortest-path horizontal travel distance, such as the distance a lift truck would drive through the aisles, and do not consider vertical travel required to access upper locations in a rack. We assume that travel takes place in the center of the aisles, both sides of a picking aisle can be accessed with negligible travel across the aisle, and the storage locations are one pallet deep. Therefore, the distance to a given pallet position is the same as the distance to a pallet position directly above, below, or across the picking aisle. A "location" then refers to a point in a given picking aisle that corresponds to two columns of pallet positions that are the same distance from the P&D point. For example, a warehouse with 21 picking aisles that are 50 pallets long, would have a total picking aisle length $N = 1050$, with 2100 pallet positions on each storage rack level. The same definition of N applies to all designs, although with fishbone, for example, the aisle lengths vary. In our experiments, we define the size of a warehouse by the total picking aisle length, N, classifying $1000 < N \leq 4000$ as "relatively large" warehouses.

Storage and retrieval requests are assumed to be independent and processed on a first-come-first-serve basis. Although strategies for sequencing storage and retrieval requests in a dual-command environment can be shown to reduce travel distances (Hwang and Schaefer, 1996), we do not address that issue in this paper; our focus is on comparing different aisle layout designs under a given operational policy.

3.1 Demand Model

Activity-based inventory analysis ranks all items in an inventory by their contribution to total demand. The demand frequency curve is a plot of ranked cumulative percent demand versus percent of inventoried items. Bender (1981) represents the demand frequency curve with the model,

$$F(x) = \frac{(1+S)x}{S+x} \qquad F(x) \geq 0 \text{ and } x \leq 1, S \geq 0 \text{ and } S + x \neq 0,$$

where $F(x)$ is the cumulative percent demand, or a percentage of total warehouse activity, and x is a fraction of the total number of items stored. $F(x)$ is therefore a cumulative distribution function defined for $x \in [0, 1]$. It is typical to find that a small percentage of the items represents a large percentage of the total demand. S is the shape parameter, which determines the skewness of the demand curve. The notation "$100x/100F(x)$" indicates that $100x$ percent of the items represents $100F(x)$ percent of the total demand. For this paper we use 20/40, 20/60 and 20/80 curves ($S = 0.6$, $S = 0.2$ and $S = 0.0667$, respectively). The random storage policy can also be represented by this model as a 20/20 curve with an arbitrarily large value of S.

We assume that N items are in inventory. The probability that a random request is for item k is p_k, where $\sum_{k=1}^{N} p_k = 1$. The Bender model is a continuous model that is discretized to determine the probabilities p_k. For each item k,

$$p_k = F\left(\frac{k}{N}\right) - F\left(\frac{k-1}{N}\right) \qquad \forall k = 1, 2, \ldots, N. \tag{1}$$

The slotting strategy then allocates N items (with their probabilities p_k) to the N storage locations in the warehouse.

Our modeling approach assumes that the turnover frequency of each item is known and constant through time. In practice, demand rates are dynamic, which leads to the need for warehouses using a turnover-based storage policy to reassign items to storage locations over time. However, we fix the demand curve over all the layouts considered to provide a consistent basis for comparison.

3.2 Travel Distance Models

Given that the N items have been allocated to storage locations, each of the N locations has a probability p_i of being visited for a storage or retrieval. Each location i is a distance d_i from the P&D point. For a warehouse with N locations, the expected single-command travel distance is

$$E[SC] = 2 \sum_{i=1}^{N} d_i p_i.$$

Let δ_{ij} represent the shortest-path travel distance between locations i and j. The expected travel-between distance is then

$$E[TB] = \sum_{i=1}^{N} \sum_{j=1}^{N} \delta_{ij} p_i p_j.$$

The expected dual-command travel distance is

$$E[DC] = E[SC] + E[TB] = 2 \sum_{i=1}^{N} d_i p_i + \sum_{i=1}^{N} \sum_{j=1}^{N} \delta_{ij} p_i p_j.$$

4 Warehouse Design

The models presented in Section 3 allow us to estimate single- and dual-command travel distances in any warehouse under a turnover-based storage policy. We use those models to compare the performance of the flying-V and fishbone layouts to traditional aisle layouts (Layouts A, B and C); we investigate single-command operations in Section 5 and dual-command operations in Section 6. The interaction of diagonal aisles with a rectangular warehouse space makes it difficult (in some cases, impossible) to compare, say, a fishbone design having N pallets with a traditional design having N pallets. We address this obstacle by constructing designs with approximately the same number of pallets, subject to aisles containing all possible pallet positions. This leads to comparisons that vary slightly in the number of pallet locations (+/- 4%). The resulting warehouses will have different total areas, because the layouts have different amounts of aisle space. We assume square pallet footprints (which include clearances) and specify the warehouse dimensions in pallets, where the center-to-center distance between adjacent picking aisles is 5 pallets, and the cross aisle width is 3 pallets.

Our goal is to compare a well-designed flying-V (or fishbone) warehouse to well-designed traditional warehouses with the same storage capacity (defined by N). In general, for a given N and operational policy (single- or dual-command), we use specific warehouse design parameters (which define the warehouse shape and aisle structure) that work well for a random storage policy. We then use these design parameters (one set for each of the five layouts) to evaluate all four demand curves (20/20, 20/40, 20/60 and 20/80). The following assumptions apply to all of the computations in this paper:

1. **Storage Capacity:** Warehouses compared directly have approximately the same N.

2. **Warehouse Shape:** We assume a rectangular warehouse shape that is optimal under a random storage policy.

3. **Aisle Structure:** The aisle structures of Layouts A and C are determined by the warehouse shape, which dictates the number and length of the picking aisles.

   Layout B. We assume one middle cross aisle, located halfway between the top and bottom cross aisles.

   Flying-V. The shape of the flying-V cross aisle is optimized for single-command operations and random storage.

   Fishbone. We use a diagonal cross aisle that extends to the upper corners of the warehouse.

4. **Slotting Strategy:** We use the distance-based slotting strategy for all warehouses.

Per Assumption 1, we do not adjust the storage capacity of the warehouse to account for the fact that dedicated storage requires more warehouse space. This is appropriate for our analysis, given that we compare different aisle layouts with the same demand curve (rather than comparing different demand curves for one layout).

Per Assumption 2, we use the warehouse shape for each layout that is optimal under random storage. For single-command travel and random storage, the optimal shape for all five layouts is approximately half as tall as it is wide (shape factor, r = height/width ≈ 0.5). This was shown by Francis (1967) for a continuous-space rectangular warehouse, by Bassan et al. (1980) for Layouts A and C, and by Pohl et al. (2009a) for Layouts A, B and C. For dual-command travel in Layouts A, B and C, the optimal number of aisles is given by (13) in Pohl et al. (2009a). The optimal shape for dual-command travel in fishbone aisles also has shape factor $r \approx 0.5$ (Pohl et al., 2009c). For a flying-V warehouse, the optimal shape for dual-command travel has not been thoroughly investigated, but $r \approx 0.5$ performs well, and therefore we use it here.

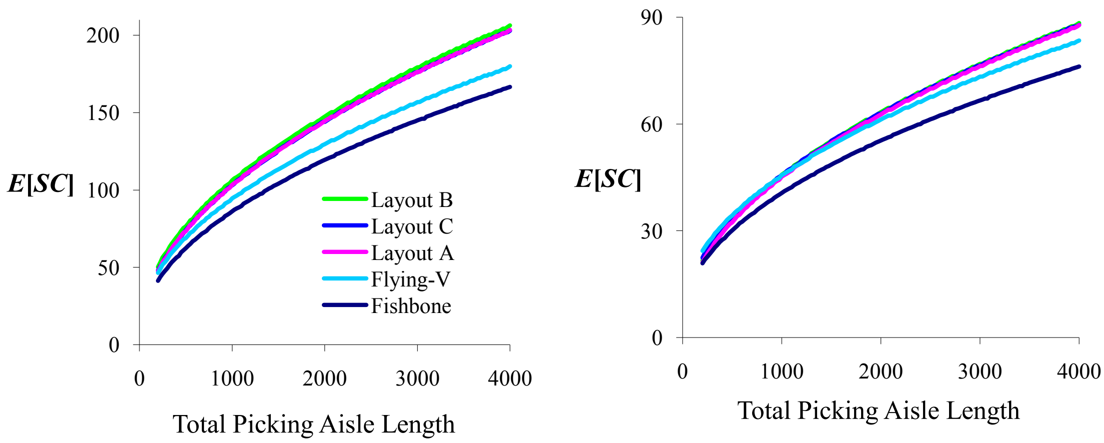
The warehouse shape that is best under random storage is not necessarily best under turnover-based storage. We investigate the impact of Assumption 2 on our results with parametric testing for three values of N, where the best shape for each demand curve (the shape that minimizes either $E[SC]$ or $E[DC]$) is determined for all five layouts and the four demand curves. The "errors" resulting from using the designs most suitable for random storage are less than 1.3% for all five layouts. The results of this analysis are in Appendix A of the online supplement to this paper.

Assumption 3 dictates a particular aisle structure for each layout. For simplicity, we assume the middle cross aisle in Layout B is halfway between the top and bottom cross aisles, although the optimal cross aisle location under turnover-based storage may be closer to the bottom cross aisle. For the flying-V, our testing has shown that $E[SC]$ can be slightly improved by changing the cross aisle shape (see Appendix B of the online supplement); although determining the optimal cross aisle shape for each warehouse size and demand curve is computationally intensive, with only small improvements in performance (a maximum of 0.38% for our test cases). The flying-V cross aisle is not as efficient for travel-between as a traditional perpendicular cross aisle (Pohl et al., 2007), and would therefore not typically be chosen for dual-command operations; however we include the results here for completeness. Under a random storage policy, the fishbone design that minimizes both single- and dual-command travel has a diagonal cross aisle that extends to the upper corners of the warehouse (Pohl et al., 2009c). Testing with three values of N indicate that when we use these design parameters for turnover-based storage, the maximum error is 0.53% (see Appendix C of the online supplement).

We use Assumptions 1-4 to develop a set of design parameters for each layout (flying-V, fishbone and Layouts A, B and C), each operational mode (single- and dual-command), and each warehouse size ($N = 200$ to 4000, in increments of 25). We then evaluate comparable designs for each demand curve (random storage, 20/40, 20/60 and 20/80) using the distance-based slotting strategy, and compare the overall performance of the layouts.

5 Single-Command Operations

We first evaluate the expected single-command travel distances for the five aisle designs and each demand curve, over a range of warehouse sizes. Figures 5(a) and 5(b) illustrate the results for random storage and the 20/80 demand curve, respectively. In Figure 5, and subsequent figures, the units for expected travel distances and the "Total Picking Aisle Length" is pallet widths. As expected, turnover-based storage reduces the expected travel distances for all layouts (note the scaling of each y-axis). For most warehouse sizes and demand skewness levels, the fishbone design is best, the flying-V design is 2nd best and the traditional warehouses perform very similarly to each other.



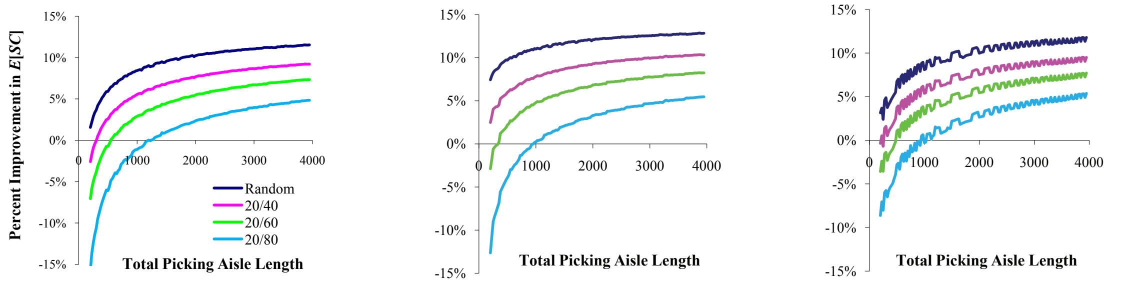
(a) Random storage (20/20)

(b) Turnover-based storage (20/80)

Figure 5: Expected single-command travel distance for random storage and 20/80 turnover-based storage: the fishbone design is always best

Figures 6(a), (b) and (c) plot the percent improvement in expected single-command travel distance in warehouses with flying-V cross aisles, compared to Layouts A, B and

C, respectively. The curves in Figure 6(c) are not smooth because as N increases, the numbers of picking aisles in the Flying-V and Layout C warehouses change discretely and independently of one another. The curves in Figures 6(a) and (b) are smooth because the flying-V warehouses have the same number of picking aisles and exactly the same values of N as the Layout A and B warehouses.

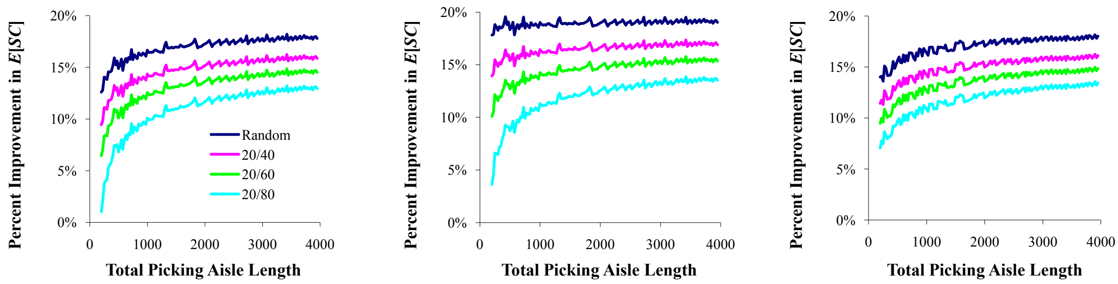


(a) Flying-V versus Layout A (b) Flying-V versus Layout B (c) Flying-V versus Layout C

Figure 6: Expected single-command travel distance performance for four demand skewness levels: flying-V cross aisle compared to traditional aisle layouts

Figure 6 shows that the percentage by which the flying-V design improves on the traditional designs decreases with increasing skewness of the demand. This result is intuitive, because the locations farthest from the P&D point are those that benefit the most from the flying-V cross aisle; as the demand skewness increases, these locations are visited less frequently. Under random storage, the flying-V design is always better than the traditional designs, but under turnover-based storage, we see “negative improvement” for some warehouses with $N \leq 1000$; i.e., expected travel distances are longer in the flying-V warehouses. For highly-skewed demand and small warehouses, the few pallet locations closest to the P&D point are visited much more frequently than others. When a flying-V cross aisle is inserted, some of these close locations are “pushed” farther away from the P&D point. With relatively large warehouses ($N > 1000$) or less skewed demand, this penalty is compensated for by the more efficient travel to other locations. These results indicate that the flying-V cross aisle is preferred over traditional layouts under a random storage policy, or for relatively large warehouses under turnover-based storage policies.

Figure 7 compares the single-command performance of fishbone aisles to the traditional layouts. As with the flying-V, the percentage by which fishbone improves on the traditional designs decreases with increasing skewness. Single-command travel distances are shorter in fishbone warehouses in all instances. For relatively large warehouses ($N > 1000$), we can expect an improvement in single-command travel distances of 10–20%.



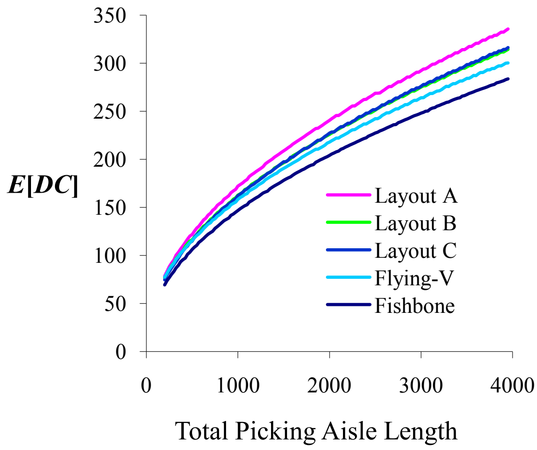
(a) Fishbone versus Layout A (b) Fishbone versus Layout B (c) Fishbone versus Layout C

Figure 7: Expected single-command travel distance performance for four demand skewness levels: Fishbone aisles compared to traditional aisle layouts

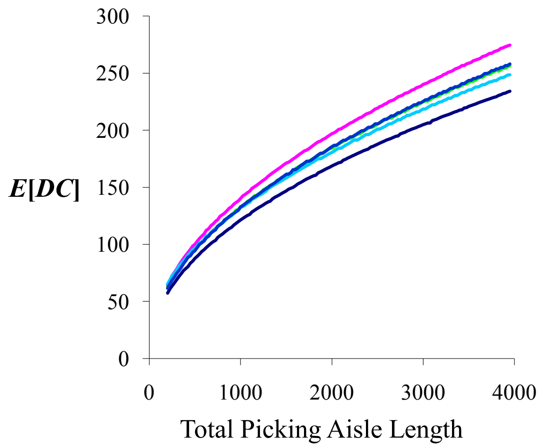
6 Dual-Command Operations

Figure 8 shows the expected dual-command travel distances for the five aisle designs for each of the four demand curves. The more skewed the demand curve, the smaller the expected travel distance for all layouts. Fishbone is the best-performing design for all warehouse sizes ($N = 200$ to 4000) and demand skewness levels, and Layout A is, in most cases, the worst-performing design.

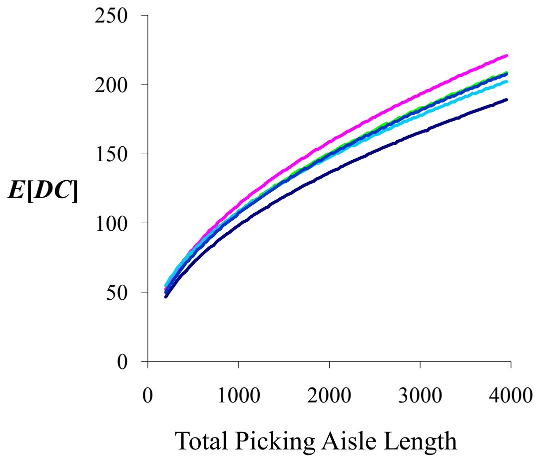
A comparison of the three traditional layouts reveals that Layouts B and C are typically preferred over Layout A for dual-command operations because the middle cross aisle generally reduces the travel-between component of the dual-command cycle (Pohl et al., 2009a). The performance of Layouts B and C is very similar for random storage, but as the demand skewness increases, Layout C is slightly better. We attribute this to the fact that the middle cross aisle in Layout B is used less as the demand skewness increases because more of the activity is in the bottom half of the warehouse. In contrast, the effectiveness of the middle



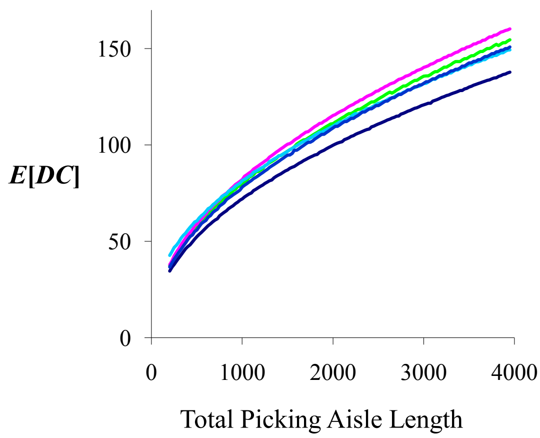
(a) Random storage (20/20)



(b) Turnover-based storage (20/40)



(c) Turnover-based storage (20/60)

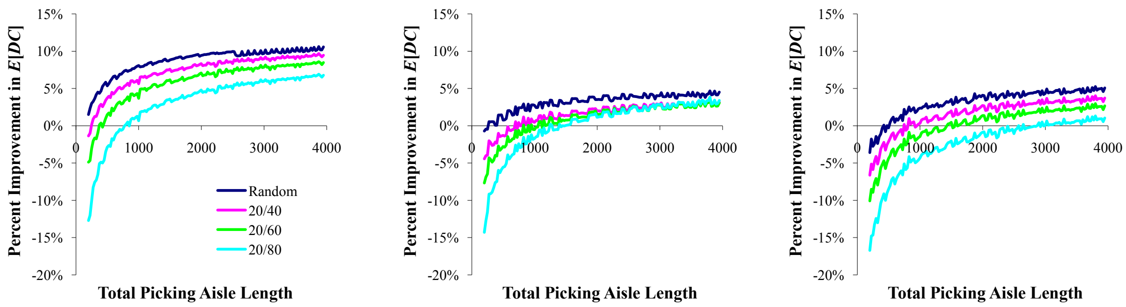


(d) Turnover-based storage (20/80)

Figure 8: Expected dual-command travel distance for four demand skewness levels

cross aisle in Layout C is unaffected by demand skewness because of symmetry relative to the P&D point. The performance of Layout B under turnover-based storage might be improved by moving the middle cross aisle closer to the P&D point. Optimally locating the middle cross aisle under turnover-based storage policies is an interesting design question, but is beyond the scope of this paper.

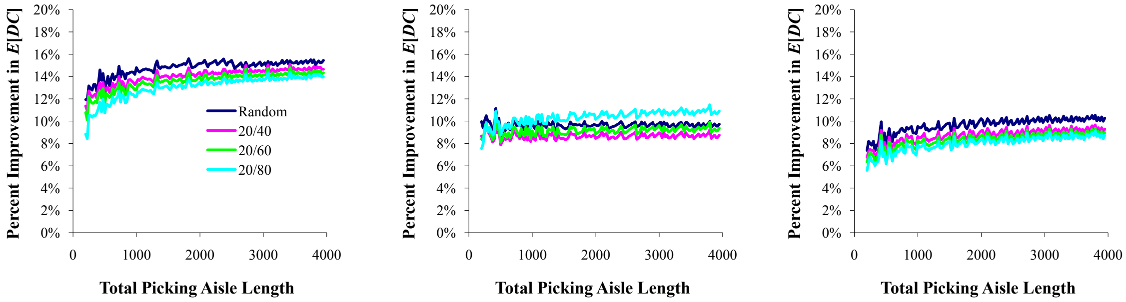
Figure 9 shows the dual-command performance of the flying-V design relative to the traditional layouts. Improvement over Layout A (Figure 9(a)) is very similar to the results for single-command travel (Figure 6(a)), where the flying-V improves on Layout A performance for all but warehouses with $N \leq 1000$ and turnover-based storage. The dual-command results for Layouts B and C are quite different than they were for single-command, in that improvement is at most 5%, and negative in a number of instances. This is because travel-between is less efficient in the flying-V warehouses (for all cases) than it is in either Layout B or C.



(a) Flying-V versus Layout A (b) Flying-V versus Layout B (c) Flying-V versus Layout C

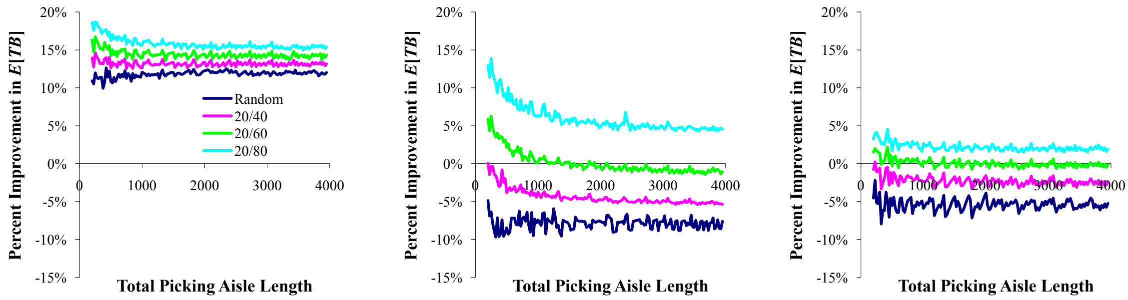
Figure 9: Expected dual-command travel distance performance for four demand skewness levels: flying-V cross aisle compared to traditional aisle layouts

Figure 10 compares the dual-command performance of fishbone aisles to the traditional layouts. The improvement over Layouts A, B and C is approximately 10–16%, 8–11% and 6–10%, respectively. The relative performance of the fishbone design for dual-command travel is much less sensitive to demand skewness, compared to the single-command results in Figure 7 (the curves are closer together). We can explain this phenomenon by examining the travel-between component of the dual-command cycle. For single-command travel, the *relative* performance of fishbone decreases with increasing skewness. However, for travel-between the reverse is true (see Figure 11), resulting in smaller differences between the four demand curves for dual-command travel distance.



(a) Fishbone versus Layout A (b) Fishbone versus Layout B (c) Fishbone versus Layout C

Figure 10: Expected dual-command travel distance performance for four demand skewness levels: fishbone aisles compared to traditional aisle layouts



(a) Fishbone versus Layout A (b) Fishbone versus Layout B (c) Fishbone versus Layout C

Figure 11: Expected travel-between distance performance for four demand skewness levels: fishbone aisles compared to traditional aisle layouts

7 Conclusions

For single-command travel, the flying-V cross aisle is better than the traditional layouts under random storage, and for almost all warehouse sizes under turnover-based storage. For warehouses with $N \leq 1000$ and highly-skewed demand, the traditional aisle layouts offer shorter expected travel distances than the flying-V. The flying-V cross aisle generally does not perform well for warehouses that use 100% dual-command operations and use a turnover-based storage policy; under these operating conditions a traditional orthogonal cross aisle (as in Layouts B and C) provides similar or better performance. The strength of the flying-V cross aisle lies in what it is designed to address: single-command travel under a random storage policy.

The fishbone design performs better than the traditional designs over a range of warehouse sizes for both single- and dual-command travel. For single-command travel, the improvement is 10–20% for relatively large warehouses. Improvement in dual-command travel distances is 6–16% across all warehouse sizes. For dual-command operations, the performance of fishbone aisles is robust with respect to storage policy. This leads to a useful result for warehouse managers who might consider changing the storage policy:

> *A fishbone warehouse that reduces expected dual-command travel distance under random storage, will offer a similar improvement under turnover-based storage.*

The design rules for fishbone warehouses that apply under a random storage policy (Pohl et al., 2009c), are also appropriate under turnover-based storage:

> *The best, or nearly-best, design is obtained (1) by choosing a warehouse shape that is approximately a square half-warehouse, and (2) by extending the diagonal cross aisle to the upper corners of the picking space.*

Our study shows that for all five aisle layouts (flying-V, fishbone and the three traditional layouts), the warehouse shape and aisle structure that performs best under random storage also performs well under turnover-based storage for both single- and dual-command operations. The practical implication is that

> *A warehouse designed for one storage policy (or demand curve) will also be a good design if the storage policy (or demand profile) changes.*

This last result is significant, as it illustrates that the warehouse is robust to changes in the relative importance of individual items over time.

Acknowledgements

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Appendices - Intended as an Online Supplement

A Rectangular Warehouse Shape under Turnover-Based Storage

Do the optimal warehouse shapes under random storage also work well under turnover-based storage? We address this question with a series of tests. We first consider Layout A and three warehouse sizes, $N = 300$, 1000 and 3000. For each size, we vary the number of picking aisles, n, to produce warehouses of various shapes and then calculate both $E[SC]$ and $E[DC]$ for each value of n. The data are shown in Table A-1 and plotted in Figure A-1. To calculate the shape factor (r = height/width), we ignore the top and bottom cross aisle space and let $r = L/an$, where L is the picking aisle length and a is the width of each picking aisle. Note that for each warehouse size, the actual N varies due to discrete changes in the number of aisles. For $N \approx 300$, the actual values of N range from 294 to 306 (a maximum variation of 2.0%).

In the comparison analyses in Sections 5 and 6, we choose the optimal number of aisles, n^*, for each warehouse size (N = 200–4000) by using the methods described by Pohl et al. (2009a), which are based on a random storage policy and continuous picking aisles (rather than a discrete number of locations in each aisle). We denote the optimal number of aisles for single-command travel as $n^*(SC)$, and for dual-command travel as $n^*(DC)$. For these three sizes ($N = 300$, 1000 and 3000), $n^*(SC)$ is indicated in Table A-1 by a †, $n^*(DC)$ is indicated by a ‡, and the corresponding shape factors are shown in Figure A-1 by a vertical dashed line. For Layout A, $n^*(SC)$ usually equals $n^*(DC)$, as it does for these three sizes. The minimum values for each travel distance are shown in bold in Table A-1.

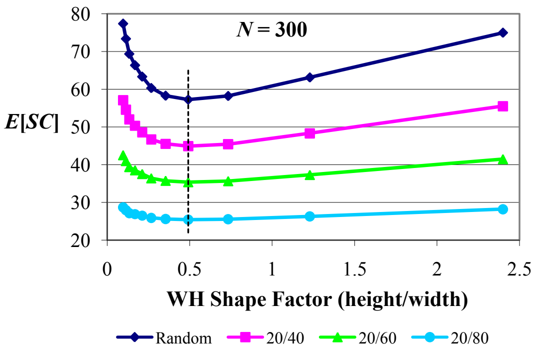
The curves in Figure A-1 all appear to be convex. For random storage, the analytical expression for $n^*(SC)$ is provably convex, and the expression for $n^*(DC)$ is convex for reasonable parameter values (N is is at least several times larger than the aisle widths) as

Table A-1: Expected single- and dual-command travel distances for Layout A over a range of warehouse shapes

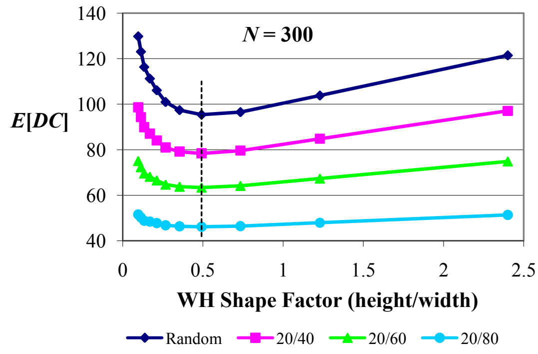
				$E[SC]$				$E[DC]$			
N	n	L	r	random	20/40	20/60	20/80	random	20/40	20/60	20/80
$N \approx 300$											
300	5	60	2.40	75.00	55.50	41.46	28.19	121.40	97.10	74.86	51.41
301	7	43	1.23	63.14	48.31	37.30	26.32	103.77	84.85	67.35	47.94
297	9	33	0.73	58.22	45.42	35.65	25.56	96.49	79.62	64.16	46.45
297	$11^{\dagger\ddagger}$	27	0.49	**57.27**	**44.91**	**35.39**	**25.45**	**95.37**	**78.41**	**63.40**	**46.15**
299	13	23	0.35	58.31	45.53	35.75	25.63	97.37	79.19	63.77	46.38
300	15	20	0.27	60.33	46.70	36.41	25.93	100.93	81.00	64.70	46.81
306	17	18	0.21	63.35	48.57	37.55	26.51	106.07	84.10	66.55	47.80
304	19	16	0.17	66.37	50.34	38.53	26.92	111.19	87.06	68.15	48.45
294	21	14	0.13	69.38	52.03	39.38	27.17	116.29	89.93	69.57	48.83
299	23	13	0.11	73.39	54.59	40.97	27.98	123.02	94.32	72.33	50.27
300	25	12	0.10	77.40	57.12	42.51	28.72	129.75	98.68	75.03	51.60
min				57.27	44.91	35.39	25.45	95.37	78.41	63.40	46.15
%diff				0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
$N \approx 1000$											
999	9	111	2.47	136.22	100.23	74.39	50.11	223.59	178.23	136.53	93.08
1001	11	91	1.65	121.27	91.01	68.95	47.62	200.09	161.65	126.14	88.18
1001	13	77	1.18	112.31	85.65	65.89	46.26	185.98	151.77	120.15	85.47
1005	15	67	0.89	107.33	82.82	64.37	45.64	178.20	146.31	116.99	84.16
1003	17	59	0.69	104.35	81.13	63.44	45.23	173.59	142.84	114.93	83.27
1007	$19^{\dagger\ddagger}$	53	0.56	**103.37**	80.63	63.22	45.17	**172.20**	141.49	114.15	83.03
1008	21	48	0.46	103.38	80.65	63.24	45.19	172.40	141.04	113.79	82.91
989	23	43	0.37	103.39	**80.45**	**62.95**	**44.90**	172.57	**140.20**	**112.84**	**82.19**
1000	25	40	0.32	105.40	81.69	63.71	45.32	176.02	142.03	113.89	82.81
999	27	37	0.27	107.41	82.82	64.32	45.57	179.45	143.68	114.65	83.10
986	29	34	0.23	109.41	83.85	64.77	45.65	182.87	145.21	115.15	83.09
992	31	32	0.21	112.42	85.66	65.84	46.17	187.93	148.17	116.83	83.90
990	33	30	0.18	115.42	87.43	66.82	46.59	192.99	151.07	118.38	84.53
1015	35	29	0.17	119.43	90.05	68.52	47.54	199.70	155.49	121.26	86.17
min				103.37	80.45	62.95	44.90	172.20	140.20	112.84	82.19
%diff				0.00%	0.23%	0.43%	0.60%	0.00%	0.91%	1.15%	1.01%
$N \approx 3000$											
2992	17	176	2.07	221.35	163.80	122.25	82.82	366.30	293.16	225.61	154.71
3002	19	158	1.66	208.37	155.87	117.64	80.76	345.35	278.40	216.48	150.51
3003	21	143	1.36	198.38	149.86	114.19	79.22	329.22	267.06	209.56	147.37
2990	23	130	1.13	190.39	145.08	111.43	77.95	316.31	257.91	203.97	144.77
3000	25	120	0.96	185.40	142.25	109.92	77.35	308.28	252.29	200.72	143.45
2997	27	111	0.82	181.41	139.96	108.66	76.80	301.87	247.63	197.97	142.26
2987	29	103	0.71	178.41	138.22	107.67	76.34	297.07	243.95	195.73	141.23
3007	31	97	0.63	177.42	137.80	107.56	76.39	295.56	242.65	195.11	141.19
3003	33	91	0.55	**176.42**	137.23	107.24	76.24	**294.03**	241.06	194.09	140.74
3010	$35^{\dagger\ddagger}$	86	0.49	176.43	137.28	107.30	76.30	294.15	240.60	193.77	140.69
2997	37	81	0.44	176.43	**137.19**	**107.19**	**76.19**	294.25	**239.91**	**193.11**	**140.29**
3003	39	77	0.39	177.44	137.79	107.53	76.36	296.00	240.46	193.31	140.44
2993	41	73	0.36	178.44	138.28	107.74	76.39	297.74	240.85	193.26	140.31
3010	43	70	0.33	180.44	139.50	108.49	76.79	301.13	242.61	196.00	140.87
3015	45	67	0.30	182.44	140.66	109.14	77.09	304.52	244.26	195.04	141.24
3008	47	64	0.27	184.45	141.76	109.70	77.29	307.90	245.84	195.69	141.43
2989	49	61	0.25	186.45	142.80	110.17	77.41	311.28	247.34	196.20	141.45
3009	51	59	0.23	189.45	144.65	111.30	77.99	316.31	250.31	197.94	142.36
3021	53	57	0.22	192.45	146.47	112.37	78.51	321.34	253.25	199.59	143.16
3025	55	55	0.20	195.45	148.26	113.39	78.97	326.37	256.15	201.17	143.85
3021	57	53	0.19	198.46	150.02	114.37	79.39	331.40	259.02	202.69	144.45
3009	59	51	0.17	201.46	151.75	115.31	79.75	336.43	261.86	204.13	144.96
2989	61	49	0.16	204.46	153.46	116.20	80.06	341.45	264.67	205.52	145.37
min				176.42	137.19	107.19	76.19	294.03	239.91	193.11	140.29
%diff				0.00%	0.06%	0.11%	0.15%	0.04%	0.29%	0.34%	0.28%

† Denotes $n^*(SC)$

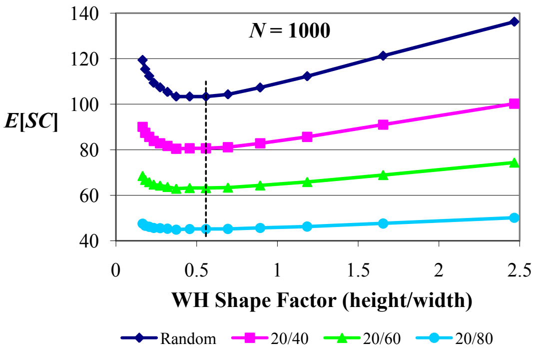
‡ Denotes $n^*(DC)$



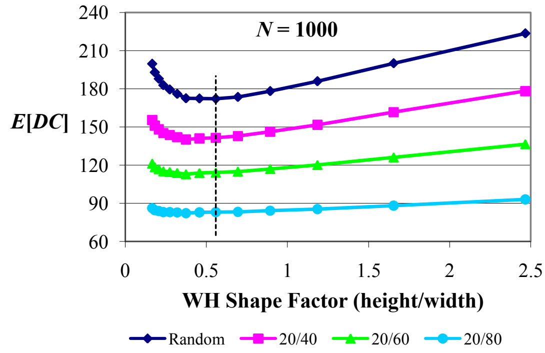
(a) $N = 300$, single-command



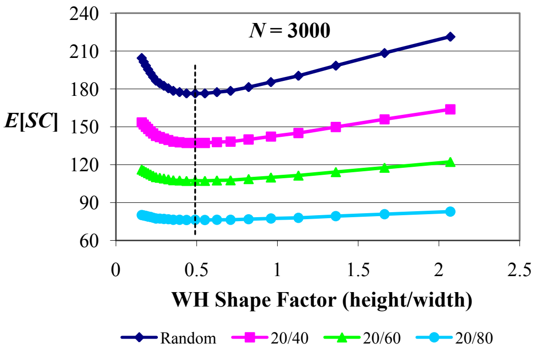
(b) $N = 300$, dual-command



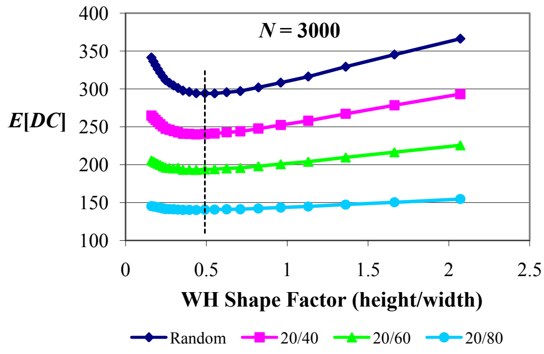
(c) $N = 1000$, single-command



(d) $N = 1000$, dual-command



(e) $N = 3000$, single-command



(f) $N = 3000$, dual-command

Figure A-1: Optimal warehouse shape for Layout A shown by Pohl et al. (2009a). For the other curves, which represent turnover-based storage, it is an open research question.

For $N \approx 300$, $n^* = 11$ for both single- and dual-command travel. Note that the minimum values for $E[SC]$ and $E[DC]$ for all demand curves occur for the design with 11 picking aisles. For this case, the storage policy does not affect our choice of design parameters.

For $N \approx 1000$, $n^* = 19$ under random storage, where $N = 1007$; but under turnover-based storage the lowest values occur at $n = 23$, where $N = 989$. Clearly the storage policy here impacts which design parameters we would choose. What is unclear is how much of that choice is influenced by the effect of going from $N = 1007$ to $N = 989$ (a decrease of 1.8%). Our analysis uses the design most suitable for random storage, therefore we have "errors" resulting from this decision (and the discrete fluctuations in N), which are 0.23%–1.15%, as shown in Table A-1.

For $N \approx 3000$, $n^* = 35$. We see the minimum values occur at $n = 33$ under random storage and $n = 37$ under turnover-based storage. The variations in N have clearly influenced these results, but the maximum error is only 0.34%.

We used the same procedure for the other four warehouse layouts (Layout B, Layout C, flying-V and fishbone). The results were similar to those for Layout A, and therefore are presented here in a slightly abbreviated form in Tables A-2–A-5. Despite the influence of discrete fluctuations in N, we can conclude that the optimal shape under turnover-based storage is often different than under random storage. However, the curves are rather flat in the neighborhood of n^*, resulting in similar performance for warehouses that vary in width by only a few aisles. The errors we introduce by our analysis approach (using the design that is best for random storage) are small, with an average error across all layouts and warehouse sizes of 0.2%.

Table A-2: Expected single- and dual-command travel distances for Layout B over a range of warehouse shapes

				$E[SC]$				$E[DC]$			
N	n	L	r	random	20/40	20/60	20/80	random	20/40	20/60	20/80
						$N \approx 300$					
301	7	43	1.23	66.21	50.05	38.25	26.72	97.82	79.70	64.44	46.99
297	$9^{‡}$	33	0.73	61.31	47.29	36.73	**26.04**	**92.39**	75.73	61.66	45.49
297	$11^{†}$	27	0.49	**60.38**	**46.96**	**36.68**	26.08	92.46	**75.45**	**61.26**	**45.21**
299	13	23	0.35	61.44	47.77	37.29	26.46	95.28	76.95	61.99	45.50
min				60.38	46.96	36.68	26.04	92.39	75.45	61.26	45.21
%diff				0.00%	0.00%	0.00%	0.16%	0.00%	0.37%	0.65%	0.62%
						$N \approx 1000$					
1003	17	59	0.69	107.40	82.98	64.51	45.72	162.84	133.38	108.72	80.66
1007	$19^{†‡}$	53	0.56	106.42	**82.58**	64.40	45.74	**162.77**	**132.90**	108.13	80.26
1008	21	48	0.46	**106.38**	82.64	64.50	45.81	164.05	133.23	107.98	80.01
989	23	43	0.37	106.46	82.62	**64.40**	**45.67**	165.41	133.32	**107.41**	**79.26**
1000	25	40	0.32	108.40	83.87	65.22	46.13	169.48	135.65	108.67	79.81
min				106.38	82.58	64.40	45.67	162.77	132.90	107.41	79.26
%diff				0.04%	0.00%	0.00%	0.15%	0.00%	0.00%	0.66%	1.25%
						$N \approx 3000$					
2987	29	103	0.71	181.44	140.06	108.73	76.81	275.28	225.33	183.58	136.08
3007	$31^{‡}$	97	0.63	180.45	139.68	108.68	76.91	275.17	224.86	183.05	135.80
3003	33	91	0.55	179.46	**139.17**	**108.43**	**76.80**	**275.04**	**224.17**	182.17	135.13
3010	$35^{†}$	86	0.49	**179.43**	139.25	108.53	76.90	276.30	224.46	181.99	134.87
2997	37	81	0.44	179.47	139.26	108.52	76.86	277.61	224.60	**181.55**	134.31
3003	39	77	0.39	180.47	139.91	108.94	77.08	280.31	225.83	181.95	134.30
2993	41	73	0.36	181.48	140.46	109.22	77.18	283.00	226.95	182.16	**134.06**
3010	43	70	0.33	183.44	141.69	110.00	77.60	287.07	229.22	183.32	134.51
min				179.43	139.17	108.43	76.80	275.04	224.17	181.55	134.06
%diff				0.00%	0.05%	0.10%	0.13%	0.04%	0.31%	0.82%	1.28%

† Denotes $n^*(SC)$

‡ Denotes $n^*(DC)$

Table A-3: Expected single- and dual-command travel distances for Layout C over a range of warehouse shapes

N	n	L	r	$E[SC]$ random	$E[SC]$ 20/40	$E[SC]$ 20/60	$E[SC]$ 20/80	$E[DC]$ random	$E[DC]$ 20/40	$E[DC]$ 20/60	$E[DC]$ 20/80
						$N \approx 300$					
304	4	76	0.26	61.00	47.41	37.25	27.09	99.96	79.56	62.87	45.25
300	$5^{\dagger}$	60	0.42	**58.00**	**45.65**	**36.25**	26.61	92.70	75.35	60.58	44.24
300	$6^{\ddagger}$	50	0.60	58.00	45.65	36.25	26.61	90.62	74.37	60.19	44.15
294	7	42	0.83	59.00	46.09	36.37	**26.57**	**90.22**	**74.15**	**60.01**	**43.99**
304	8	38	1.05	62.00	47.98	37.57	27.23	93.38	76.50	61.73	45.09
min				58.00	45.65	36.25	26.57	90.22	74.15	60.01	43.99
%diff				0.00%	0.00%	0.00%	0.15%	0.44%	0.30%	0.29%	0.37%
						$N \approx 1000$					
1008	9	112	0.40	104.00	81.15	63.72	45.81	167.28	135.21	108.16	78.46
1000	$10^{\dagger}$	100	0.50	103.00	80.51	63.31	45.57	163.19	132.87	106.88	77.88
990	11	90	0.61	**103.00**	**80.39**	**63.15**	**45.42**	**160.87**	**131.51**	**106.09**	**77.47**
1008	$12^{\ddagger}$	84	0.71	105.00	81.70	64.02	45.93	162.16	132.72	107.13	78.25
988	13	76	0.86	106.00	82.05	64.02	45.76	161.61	132.20	106.62	77.80
			min	103.00	80.39	63.15	45.42	160.87	131.51	106.09	77.47
			%diff	0.00%	0.14%	0.25%	0.33%	0.80%	0.91%	0.97%	0.99%
						$N \approx 3000$					
3008	16	188	0.43	177.00	137.67	107.63	76.67	283.82	229.33	183.28	132.55
2992	$17^{\dagger}$	176	0.48	**176.00**	137.02	107.20	76.41	279.62	226.88	181.92	131.92
2988	18	166	0.54	176.00	**136.99**	**107.16**	**76.37**	277.22	225.60	181.29	**131.69**
3002	19	158	0.60	177.00	137.63	107.57	76.61	276.64	225.54	181.47	131.96
3000	$20^{\ddagger}$	150	0.67	178.00	138.17	107.85	76.72	276.05	225.30	181.41	131.98
2982	21	142	0.74	179.00	138.61	107.98	76.68	**275.45**	**224.90**	**181.09**	131.75
2992	22	136	0.81	181.00	139.79	108.67	77.02	276.68	225.83	181.77	132.19
			min	176.00	136.99	107.16	76.37	275.45	224.90	181.09	131.69
			%diff	0.00%	0.02%	0.03%	0.05%	0.22%	0.18%	0.17%	0.22%

† Denotes $n^*(SC)$
‡ Denotes $n^*(DC)$

Table A-4: Expected single- and dual-command travel distances for flying-V over a range of warehouse shapes

N	n	L	r	$E[SC]$ random	$E[SC]$ 20/40	$E[SC]$ 20/60	$E[SC]$ 20/80	$E[DC]$ random	$E[DC]$ 20/40	$E[DC]$ 20/60	$E[DC]$ 20/80
						$N \approx 300$					
297	9	33	0.73	56.26	45.50	37.01	28.22	93.08	78.73	65.29	50.05
297	$11^{\dagger\ddagger}$	27	0.49	**55.17**	**44.93**	**36.74**	**28.14**	**91.90**	**77.43**	**64.44**	**49.64**
299	13	23	0.35	56.23	45.55	37.15	28.40	94.05	78.23	64.79	49.80
min				55.17	44.93	36.74	28.14	91.90	77.43	64.44	49.64
%diff				0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
						$N \approx 1000$					
1003	17	59	0.69	95.63	76.67	61.62	45.70	159.49	133.47	109.47	82.04
1007	$19^{\dagger\ddagger}$	53	0.56	**94.58**	76.16	61.36	45.61	**158.32**	132.29	108.69	81.64
1008	21	48	0.46	94.67	76.18	61.40	45.70	158.83	131.96	108.34	81.49
989	23	43	0.37	95.06	**76.11**	**61.18**	**45.45**	159.66	**131.49**	**107.59**	**80.82**
1000	25	40	0.32	97.33	77.45	62.05	46.02	163.68	133.68	108.92	81.63
min				94.58	76.11	61.18	45.45	158.32	131.49	107.59	80.82
%diff				0.00%	0.06%	0.30%	0.36%	0.00%	0.60%	1.01%	1.00%
						$N \approx 3000$					
3007	31	97	0.63	157.65	125.82	100.38	73.38	264.75	219.96	179.13	132.52
3003	33	91	0.55	**156.82**	**125.34**	100.12	73.29	**263.71**	218.79	178.35	132.20
3010	$35^{\dagger\ddagger}$	86	0.49	156.89	125.35	100.11	73.28	264.10	218.44	177.99	131.93
2997	37	81	0.44	157.25	125.39	**100.05**	**73.18**	264.86	**218.15**	**177.53**	**131.54**
3003	39	77	0.39	158.37	126.05	100.39	73.34	267.22	219.03	177.87	131.66
min				156.82	125.34	100.05	73.18	263.71	218.15	177.53	131.54
%diff				0.05%	0.01%	0.06%	0.14%	0.15%	0.13%	0.26%	0.30%

† Denotes $n^*(SC)$
‡ Denotes $n^*(DC)$

Table A-5: Expected single- and dual-command travel distances for fishbone over a range of warehouse shapes

N	n'	h	b	r	$E[SC]$ random	20/40	20/60	20/80	$E[DC]$ random	20/40	20/60	20/80
$N \approx 300$												
298	11	35.90	35.90	0.72	50.97	40.82	32.86	24.57	84.98	70.17	56.82	41.73
298	$13^{\dagger\ddagger}$	29.40	29.40	0.49	**49.36**	**39.99**	**32.43**	**24.40**	**83.19**	**68.89**	**56.04**	**41.37**
297	15	24.85	24.85	0.36	50.17	40.42	32.64	24.47	85.27	69.84	56.52	41.55
min					49.36	39.99	32.43	24.40	83.19	68.89	56.04	41.37
%diff					0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
$N \approx 1000$												
998	19	61.38	61.38	0.68	87.78	69.92	55.76	40.84	148.37	122.52	99.08	72.48
999	$21^{\ddagger}$	54.92	54.92	0.55	86.20	69.07	**55.29**	**40.62**	**146.72**	121.31	98.37	72.18
1000	$23^{\dagger}$	49.69	49.69	0.45	**86.15**	**69.07**	55.30	40.63	147.23	**121.28**	**98.25**	**72.09**
1000	25	45.38	45.38	0.38	87.20	69.65	55.64	40.80	149.63	122.49	98.98	72.49
				min	86.15	69.07	55.29	40.62	146.72	121.28	98.25	72.09
				%diff	0.00%	0.00%	0.01%	0.03%	0.00%	0.02%	0.12%	0.13%
$N \approx 3000$												
2996	33	99.26	99.26	0.62	146.44	116.34	92.29	66.83	249.31	205.91	166.32	121.25
3000	$35^{\ddagger}$	93.23	93.23	0.55	145.29	115.73	91.95	66.67	**248.19**	205.00	165.75	120.97
3000	$37^{\dagger}$	87.90	87.90	0.49	**145.00**	**115.58**	**91.86**	**66.62**	248.27	**204.72**	**165.49**	**120.77**
3002	39	83.18	83.18	0.44	145.48	115.86	92.04	66.72	249.86	205.38	165.88	121.09
				min	145.00	115.58	91.86	66.62	248.19	204.72	165.49	120.77
				%diff	0.00%	0.00%	0.00%	0.00%	0.00%	0.14%	0.16%	0.16%

n' is the number of vertical aisles
† Denotes $n^*(SC)$
‡ Denotes $n^*(DC)$

B Flying-V Cross Aisle Optimized for Single-Command Travel under Turnover-Based Storage

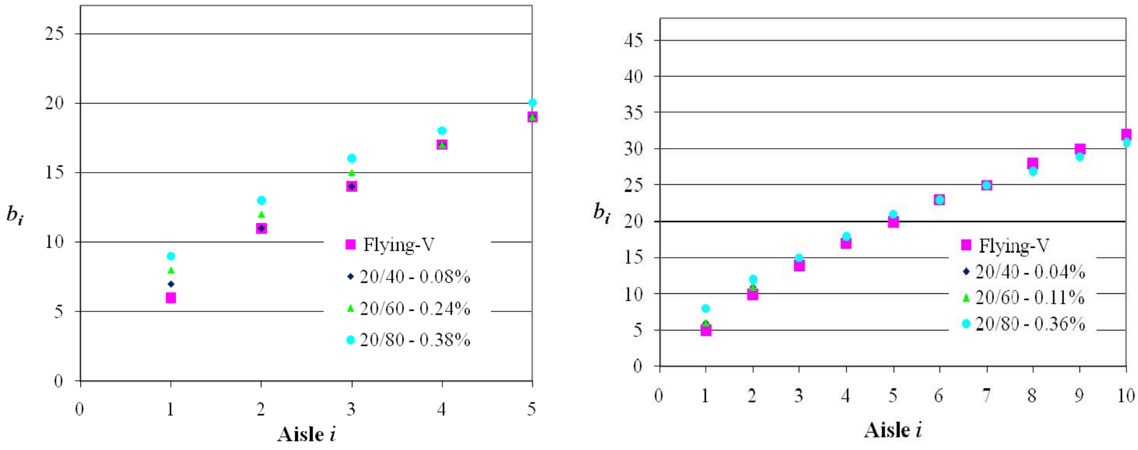
The shape of the flying-V cross aisle is optimized for single-command operations and random storage. Is this also the optimal cross aisle shape for single-command operations and turnover-based storage? To investigate this question, we choose a warehouse with 11 picking aisles of length 27 ($N = 297$). We solve for the flying-V cross aisle in accordance with Gue and Meller (2009), and then for each demand curve (20/40, 20/60 and 20/80), we attempt to find a cross aisle shape that reduces $E[SC]$. Let b_i be the number of pallet locations below cross aisle i, and $27-b_i$ be the number of locations above cross aisle i, where $i = 1, ..., 5$. (We assume that the center aisle is aisle 0, $b_0 = 0$ and the warehouse is symmetric; so we consider only the right half of the warehouse.) The values of b_i that correspond to the flying-V cross aisle are 6, 11, 14, 17 and 19, as indicated in Table B-1.

Table B-1: Cross aisle shapes under turnover-based storage, $N = 297$

	Number of Better Solutions Found	Best Solution b_1	b_2	b_3	b_4	b_5	Improvement
Flying-V		6	11	14	17	19	
20/40	42	7	11	14	17	19	0.08%
20/60	1,561	8	12	15	17	19	0.24%
20/80	20,255	9	13	16	18	20	0.38%

Because there is a discrete number of locations in each aisle and the cross aisle would not cut a location in half, there is a finite set of values for b_1–b_5. The possible number of solutions is L^n, where L is the length of the picking aisles and n is the number of picking aisles in one half of the warehouse ($27^5 = 14,348,907$). Given the large number of possible solutions, we reduce the solution space by requiring that the cross aisle have a non-negative slope between picking aisles; i.e, $b_{i+1} \geq b_i$. The results of the search are summarized in Table B-1 and illustrated in Figure 1(a). With the 20/40 demand curve, there are 24 solutions that improve upon the flying-V cross aisle, with the best of those showing only a 0.08%

improvement. The maximum improvement is 0.38% for the 20/80 demand curve.



(a) $N = 297$ (b) $N = 1008$

Figure B-1: Flying-V under turnover-based storage

We also consider a larger warehouse, with 21 picking aisles, each of length 48 ($N = 1008$). The results are shown in Figure 1(b) and Table B-2. The maximum improvement is again for the 20/80 demand curve and is only 0.36%.

Table B-2: Cross aisle shapes under turnover-based storage, $N = 1008$

	Number of Better Solutions Found	Best Solution b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	Improvement
Flying-V		5	10	14	18	21	24	26	28	30	32	
20/40	252	6	11	15	18	21	23	25	27	29	31	0.04%
20/60	45,875	6	11	15	18	21	23	25	27	29	31	0.11%
20/80	4,196,734	8	12	15	18	21	23	25	27	29	31	0.36%

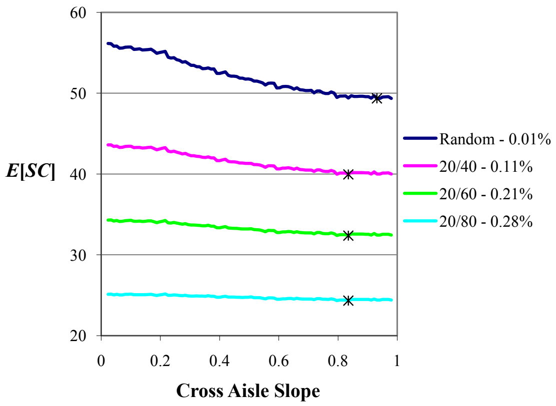
Based on these results, we conclude that while the flying-V cross aisle is not optimally shaped for turnover-based storage, it is a good design that yields expected travel distances close to the best solutions we found with our search procedure. The computational expense of finding the optimally-shaped cross aisle is not justified, given the small improvements we have seen. Using the flying-V cross aisle that is optimized for random storage is also a conservative approach, because it yields less-than-optimal results under turnover-based storage.

C Fishbone Aisles under Turnover-Based Storage

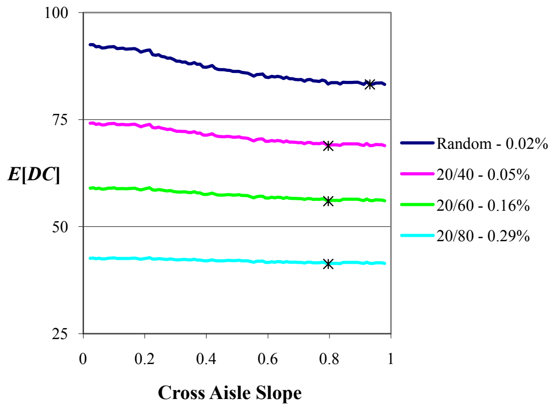
For fishbone warehouses that are shaped for single- and dual-command travel ($r \approx 0.5$), the optimal slope for the diagonal cross aisle is at (or near) its maximum value; i.e., the cross aisle extends to the upper corners of the warehouse. This statement applies to random storage, but does it also apply to turnover-based storage? To investigate this question, we consider three warehouse sizes ($N = 300$, 1000 and 3000), all with $r \approx 0.5$. For each warehouse size, we consider 100 slope values between the minimum slope of zero and the maximum slope that occurs when the diagonal cross aisle meets the upper corners of the warehouse. As the slope of the diagonal aisle increases, the number and length of the picking aisles changes discretely and the values of N vary slightly (as we saw when we varied warehouse shape in Appendix A). For instance, for $N \approx 300$, the actual values of N range from 295 to 302.

The resulting values of $E[SC]$ and $E[DC]$ for the three warehouse sizes and all demand curves are shown in Figure C-1. Note that all curves are relatively flat on the right side (particularly for 20/80), and the minimum values, indicated with an asterisk (*), do not usually occur at the maximum slope. The only instance in which the maximum slope results in the best performance is for $E[DC]$ at $N = 3000$ (Figure 1(f)). For the other curves, we indicate in the legends the percent difference between the minimum values and the values at the maximum slope; i.e., our error by assuming the maximum slope is best. The variation in the value of N contributes to this "error." For instance, in Figures 1(a) and (b), the warehouses that have the minimum values under turnover-based storage (20/40, 20/60 and 20/80) have $N = 295$, while the warehouse with the maximum slope has $N = 298$. In Figures 1(c) and (d), the warehouse that results in the minimum values has $N = 991$, while the warehouse with the maximum slope has $N = 999$.

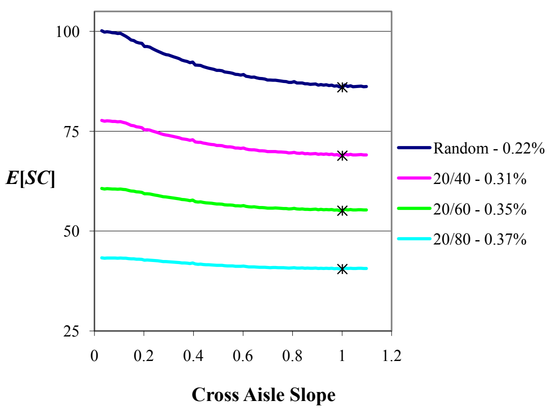
For all the results in Figure C-1, the maximum error is 0.53%. Because our approach yields slightly less-than-optimal results for the fishbone design, we accept this as a conservative approach to evaluating it under turnover-based storage.



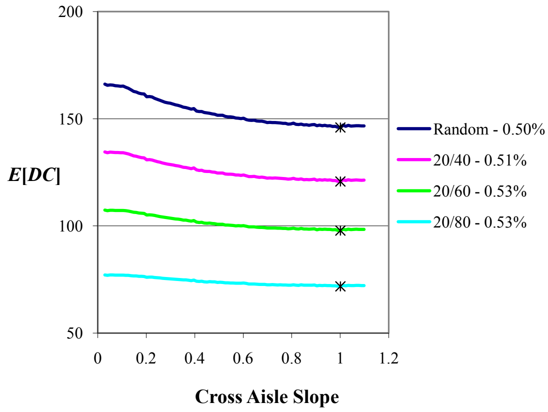
(a) $N = 300$, single-command



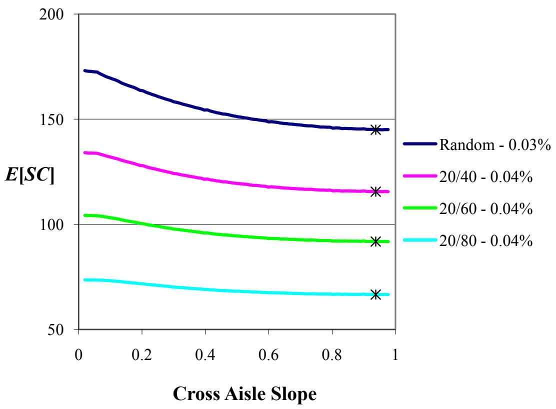
(b) $N = 300$, dual-command



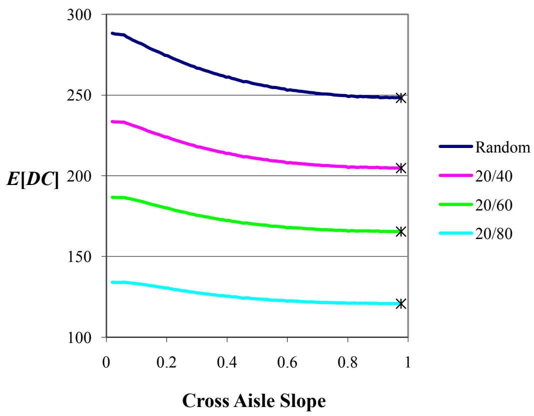
(c) $N = 1000$, single-command



(d) $N = 1000$, dual-command



(e) $N = 3000$, single-command



(f) $N = 3000$, dual-command

Figure C-1: Optimal cross aisle slope for fishbone aisles